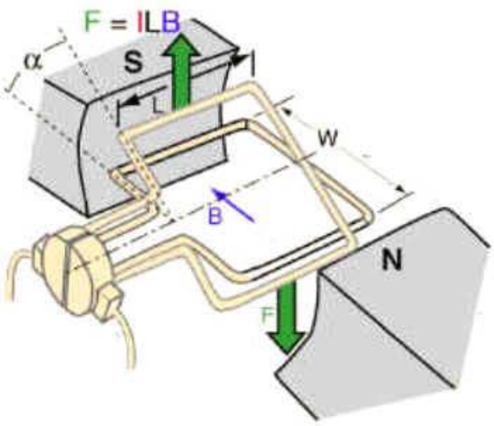
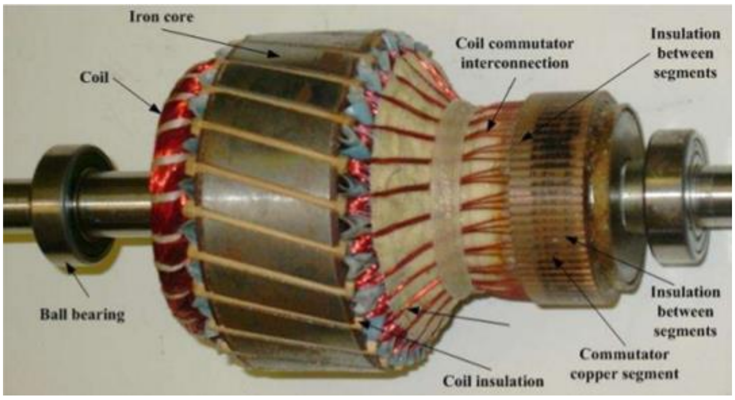
# CHAPTER # 1 THREE-PHASE WINDING, EMF'S AND MMF'S

## 1. Introduction standard

A.C. generators or alternators (as they are usually called) operate on the same fundamental principles of electromagnetic induction as D.C. generators. They also consist of an armature winding and a magnetic field. But there is one important difference between the two. Whereas in D.C. generators, the ***armature rotates*** and the field system is ***stationary***, the arrangement in alternators is just the reverse. In their case, standard construction consists of armature winding mounted on a stationary element called ***stator*** and field windings on a rotating element called rotor. The details of construction are shown in Fig. 1.



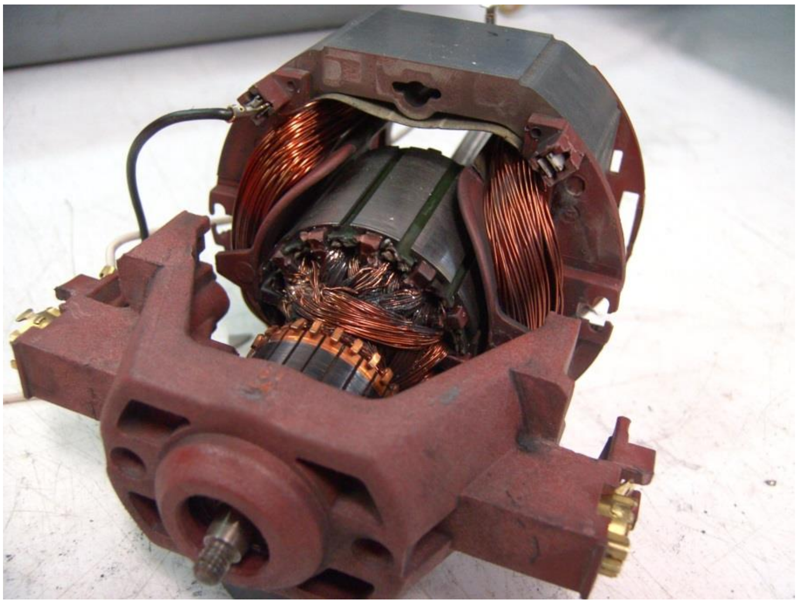


Fig. 1, Fundamental construction of DC generator

- Faraday's law:

  A moving conductor cutting the lines of force (flux) of a constant magnetic field has a voltage induced in it.

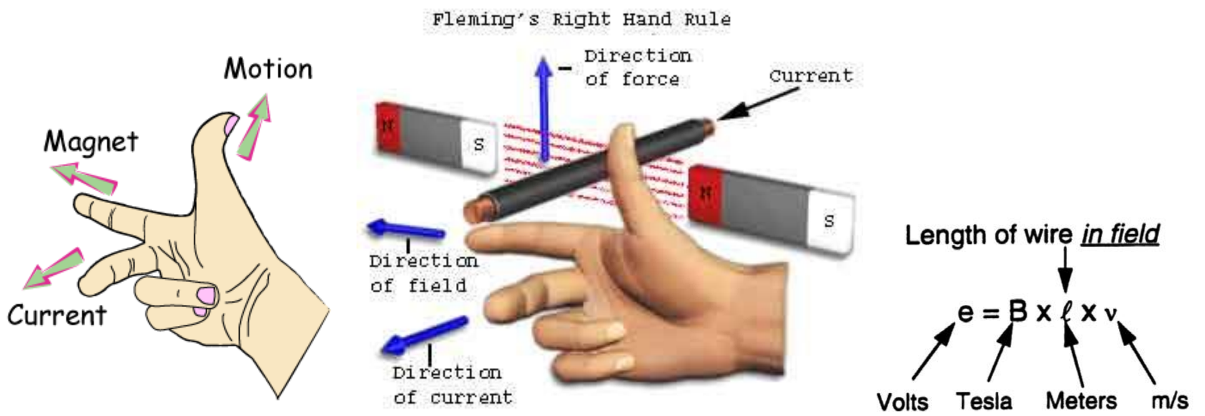



Fig. 2, Fleming right hand rule

**Fleming right hand rule** is used to determine the direction of the induced emf. If the thumb finger represents the force (F) and the forefinger represents the flux (B), then middle finger represents the induced voltage (*generator action*)

- Ampere-Biot-Savart law in its simplest form can be seen as the "reverse" of Faraday's law. While Faraday predicts a voltage induced in a conductor moving across a magnetic field, Ampere-Biot-Savart law establishes that a force is generated on a current carrying conductor located in a magnetic field.

**Fleming Left-hand rule** is used to determine the direction of the mechanical force. If the middle finger represents the current (I) and the forefinger represents the flux (B), then thumb represents the force (F) (*motor action*)

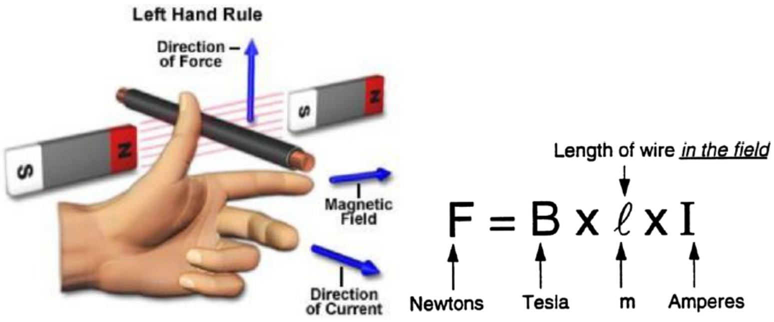



Fig. 3, Fleming left hand rule

Magnetic fields are the **fundamental mechanism by which energy is converted** from one form to another in motors, generators and transformers.

Certain materials found in nature exhibit a tendency to attract or repeal each other. These materials, called *magnets*, are also called *ferromagnetic*.

Magnets always have two poles: one called *north*; the other called *south*. In the region surrounding a permanent magnet there exists a magnetic field, which can be represented by magnetic flux lines similar to electric flux lines. Magnetic flux lines, however, do not have origins or terminating points as do electric flux lines but exist in continuous loops, as shown in Fig. 4. The symbol for magnetic flux is the Greek letter Φ (phi).

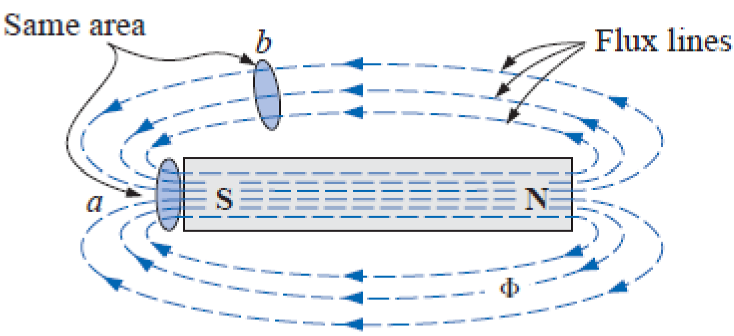



Fig. 4, Flux distribution for a permanent magnet.

The magnetic flux lines radiate from the north pole to the south pole, returning to the north pole through the metallic bar. The strength of a magnetic field in a particular region is directly related to the density of flux lines in that region. In Fig. 2, the magnetic field strength at point *a* is twice that at *b* since twice as many magnetic flux lines are associated with the perpendicular plane at a than at b. this means, the strength of permanent magnets is always stronger near the poles.

The continuous magnetic flux line will strive to occupy as small an area as possible. This will result in magnetic flux lines of minimum length between the like poles, as shown in Fig. 5.

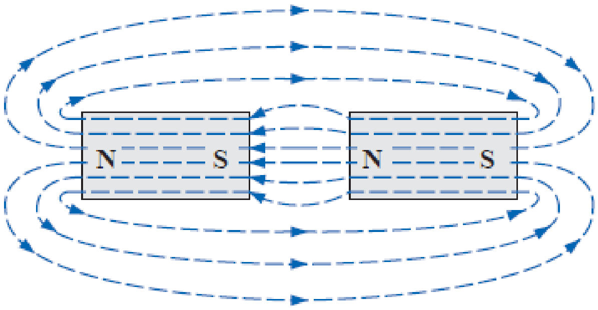



Fig. 5, Flux distribution for two adjacent, opposite poles.

If unlike poles of two permanent magnets are brought together, the magnets will attract, and the flux distribution will be as shown in Fig. 5. If like poles are brought together, the magnets will repel, and the flux distribution will be as shown in Fig. 6.

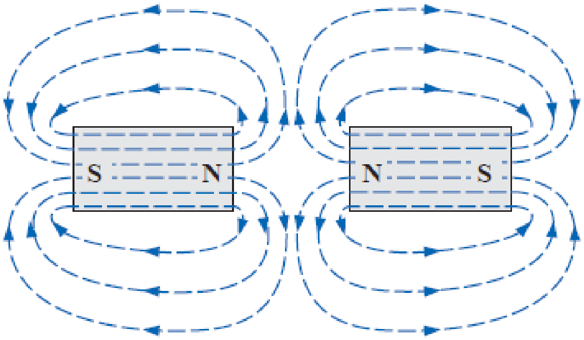



Fig. 6, Flux distribution for two adjacent, like poles.

a magnetic field (represented by concentric magnetic flux lines, as in Fig. 7) is present around every wire that carries an electric current. The direction of the magnetic flux lines can be found simply by placing the thumb of the *right* hand in the direction of current flow and noting the direction of the fingers. (This method is commonly called the *right-hand rule*.) If the conductor is wound in a single-turn coil (Fig. 8), the resulting flux will flow in a common direction through the center of the coil. A coil of more than one turn would produce a magnetic field that would exist in a continuous path through and around the coil (Fig. 9). The flux distribution of the coil is quite similar to that of the permanent magnet, and this is called electromagnet.

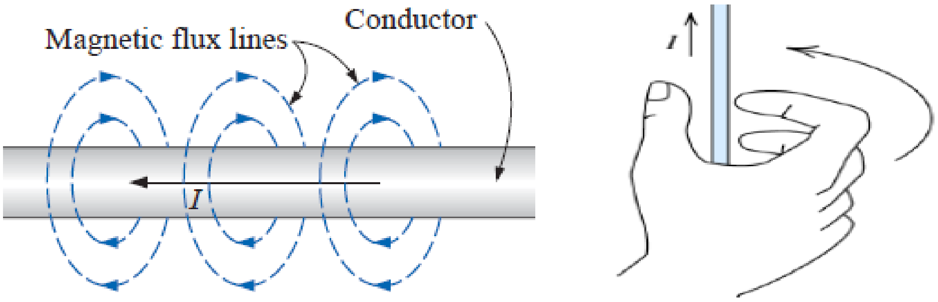



Fig. 7, Magnetic flux lines around a current-carrying conductor.

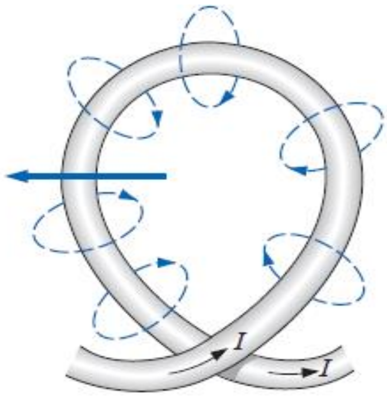



Fig. 8, Flux distribution of a single-turn coil.

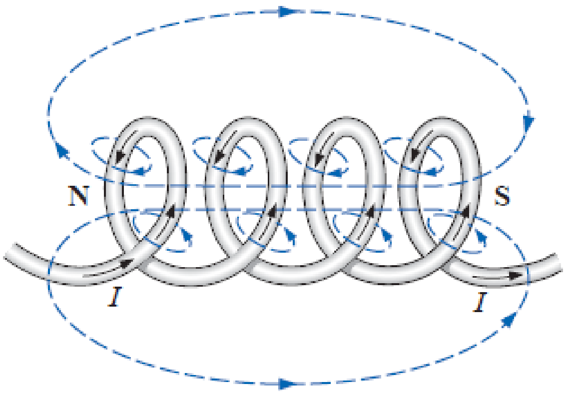



Fig. 9, Flux distribution of a current-carrying coil.

The direction of flux lines can be determined for the electromagnet by placing the fingers of the right hand in the direction of current flow around the core. The thumb will then point in the direction of the north pole of the induced magnetic flux, as demonstrated in Fig. 10. The cross and dot refer to the tail and head of the arrow, respectively.

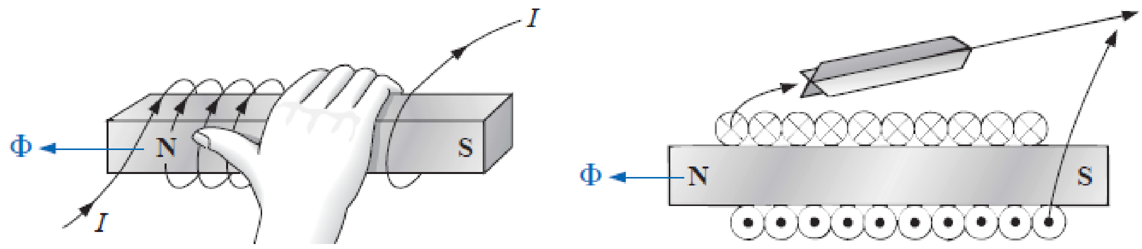



Fig. 10, Determining the direction of flux for an electromagnet

The areas of application for electromagnetic effects are shown in Fig. 11.

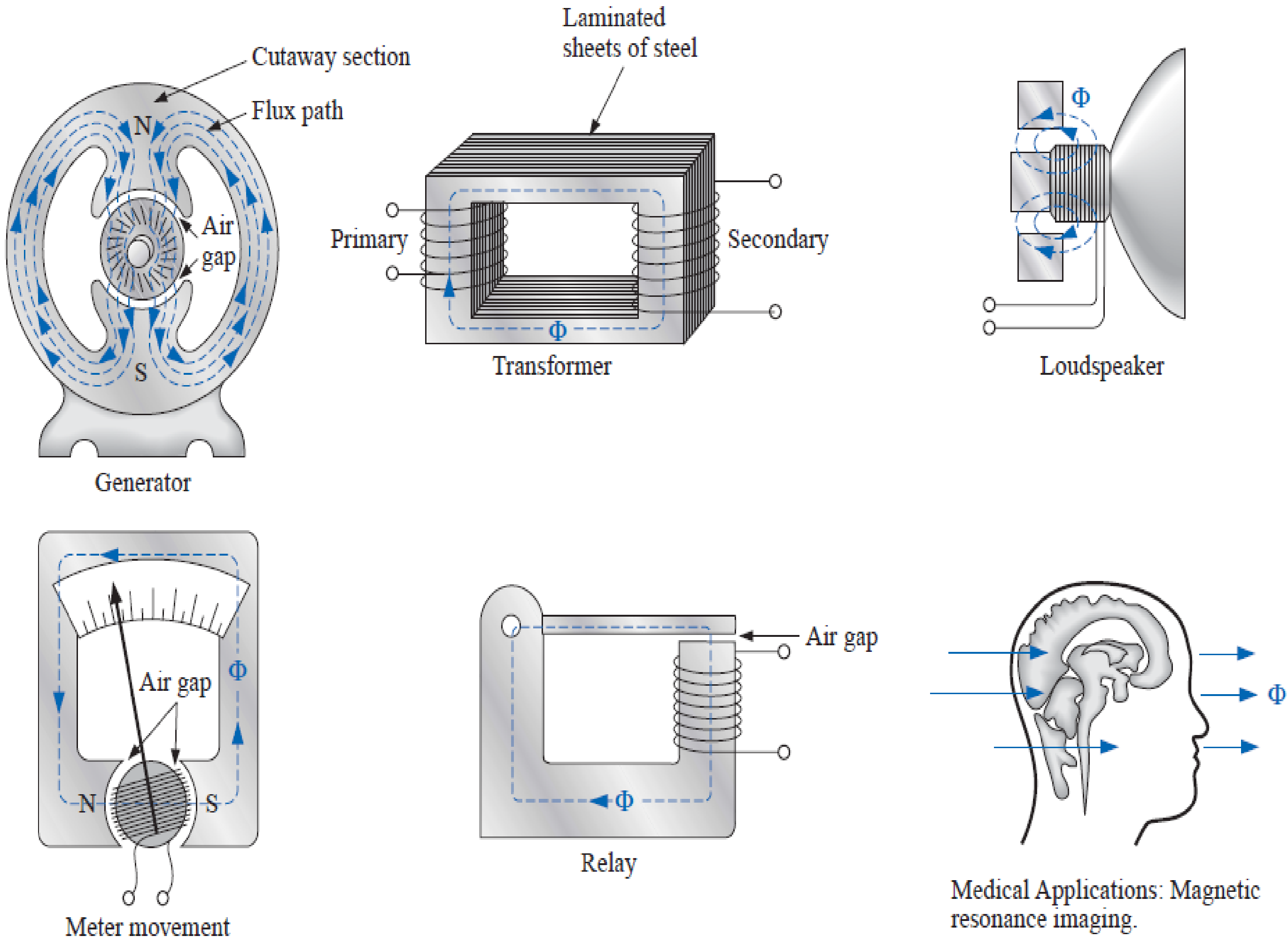



Fig. 11, Some areas of application of magnetic effects.

In the SI system of units, magnetic flux is measured in *webers*. The number of flux lines per unit area is called the **flux density,** is denoted by the capital letter *B*, and is measured in *teslas*. Its magnitude is determined by the following equation:

$$B = \frac{\Phi}{A}$$

$B$ = teslas (T)
$\Phi$ = webers (Wb)
$A$ = square meters ($m^2$)

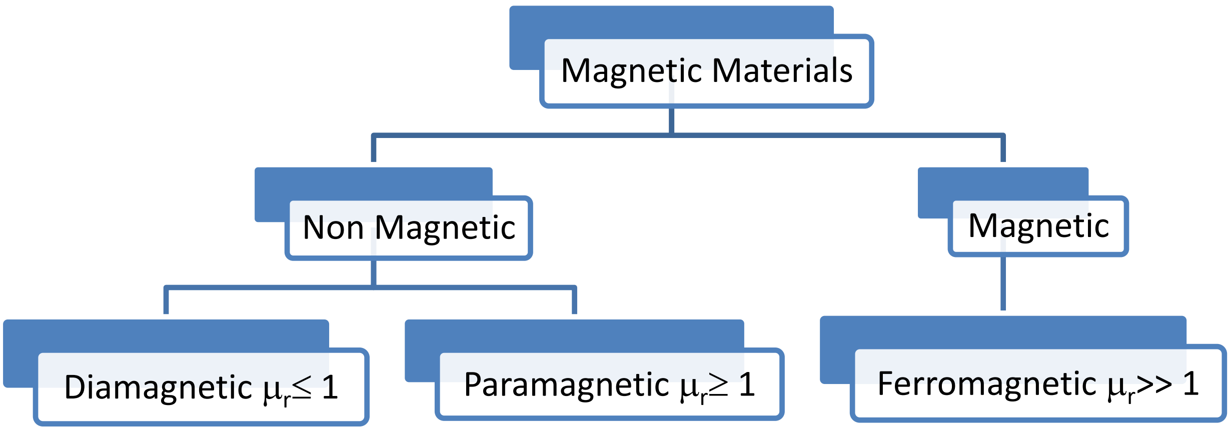
If cores of different materials with the same physical dimensions are used in the electromagnet, the strength of the magnet will vary in accordance with the core used. This variation in strength is due to the greater or lesser number of flux lines passing through the core. Materials in which flux lines can be set up are said to be *magnetic* and to have *high permeability*. The **permeability** ($\mu$) of a material, therefore, is a measure of the ease with which magnetic flux lines can be established in the material. It is similar in many respects to conductivity in electric circuits. The permeability of free space $\mu_o$ (vacuum) is:

$$\mu_o = 4\pi \times 10^{-7} \quad \frac{Wb}{At.m}$$

The ratio of the permeability of a material to that of free space is called its **relative permeability** ($\mu_r$)**;** that is,

$$\mu_r = \frac{\mu}{\mu_o}$$

According to the relative permeability, magnetic materials are classified to:




the permeability of all nonmagnetic materials, such as copper, aluminum, wood, glass, and air, is the same as that for free space. Materials that have permeabilities slightly less than that of free space are said to be **diamagnetic,** and those with permeabilities slightly greater than that of free space are said to be **paramagnetic.** Magnetic materials, such as iron, nickel, steel, cobalt, and alloys of these metals, have permeabilities hundreds and even thousands of times that of free space. Materials with these very high permeabilities are referred to as **ferromagnetic.**

The resistance of a material to the flow of charge (current) is determined for electric circuits by the equation

$$R = \rho \frac{l}{A} \qquad \text{(ohms, } \Omega\text{)}$$

The reluctance of a material to the setting up of magnetic flux lines in the material is determined by the following equation:

$$\mathscr{R} = \frac{l}{\mu A} \qquad \text{(rels, or At/Wb)}$$

where $\Re$ is the reluctance, $l$ is the length of the magnetic path, and A is the cross-sectional area. Note that the reluctance is inversely proportional to the area and directly proportional to the length. The reluctance, however, is inversely proportional to the permeability. Obviously, therefore, materials with high permeability, such as the ferromagnetics, have very small reluctances and will result in an increased measure of flux through the core. There is no widely accepted unit for reluctance, although the rel and the At/Wb are usually applied.

The Ohm's low of magnetic circuits is given by:

$$\Phi = \frac{\mathscr{F}}{\mathscr{R}}$$

The magnetomotive force MMF (F) is proportional to the product of the number of turns around the core (in which the flux is to be established) and the current through the turns of wire (Fig.12). In equation form, $\mathscr{F} = NI$

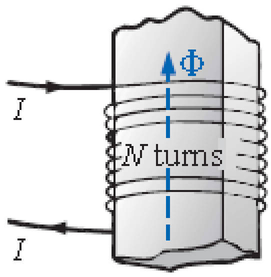



Fig. 12, Defining the components of a magnetomotive force.

The magnetomotive force per unit length is called the magnetizing force (H). In equation form,

$$H = \frac{\mathcal{F}}{l} \qquad \text{(At/m)}$$

Substituting with the value of the MMF,

$$H = \frac{NI}{l} \qquad \text{(At/m)}$$

For the magnetic circuit shown in Fig. 13, if NI=40 At and the length of the core = 0.2 m, then the magnetizing force H = 40/0.2 = 200 At/m

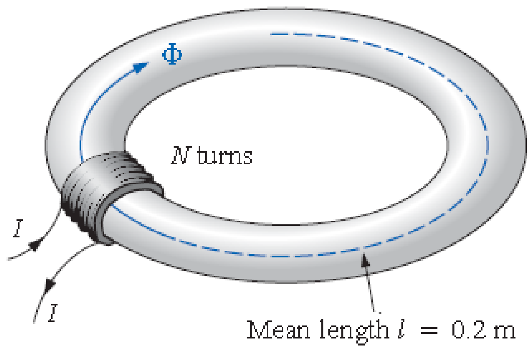



Fig. 13, Defining the magnetizing force of a magnetic circuit.

The magnetizing force also depend on the core material, therefore,

$$B = \mu H$$

Where B is the magnetic flux density Wb/m$^2$ (Tesla)

Substituting H by its value,

$$B = \mu \frac{Ni}{l_c}$$

But the flux (ϕ) produced in the = B×A, then

$$\phi = Ni \frac{\mu A}{l_c}$$

The flow of magnetic flux induced in the ferromagnetic core can be made analogous to an electrical circuit, as shown in Fig. 14, hence the name magnetic circuit is used.

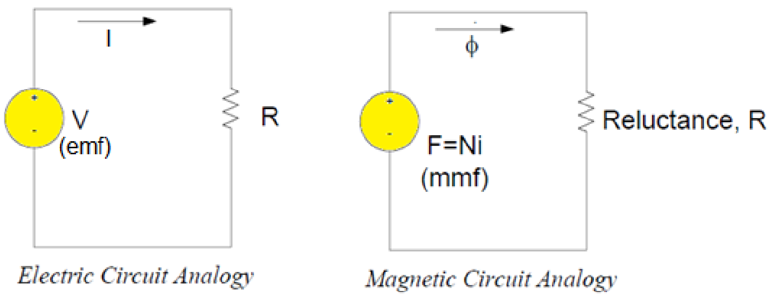



Fig. 14, Magnetic circuit analogy

Referring to the magnetic circuit analogy, F is denoted as magnetomotive force (mmf) which is similar to electromotive force in an electrical circuit (emf). Therefore, we can safely say that F is the prime mover or force which pushes magnetic flux around a ferromagnetic core at a value of Ni (refer to ampere's law).

Since V = IR (electric circuit), then F=ϕ $\Re$

$$Ni = \phi \frac{l_c}{\mu A}$$

Then the reluctance ($\Re$) is defined as:

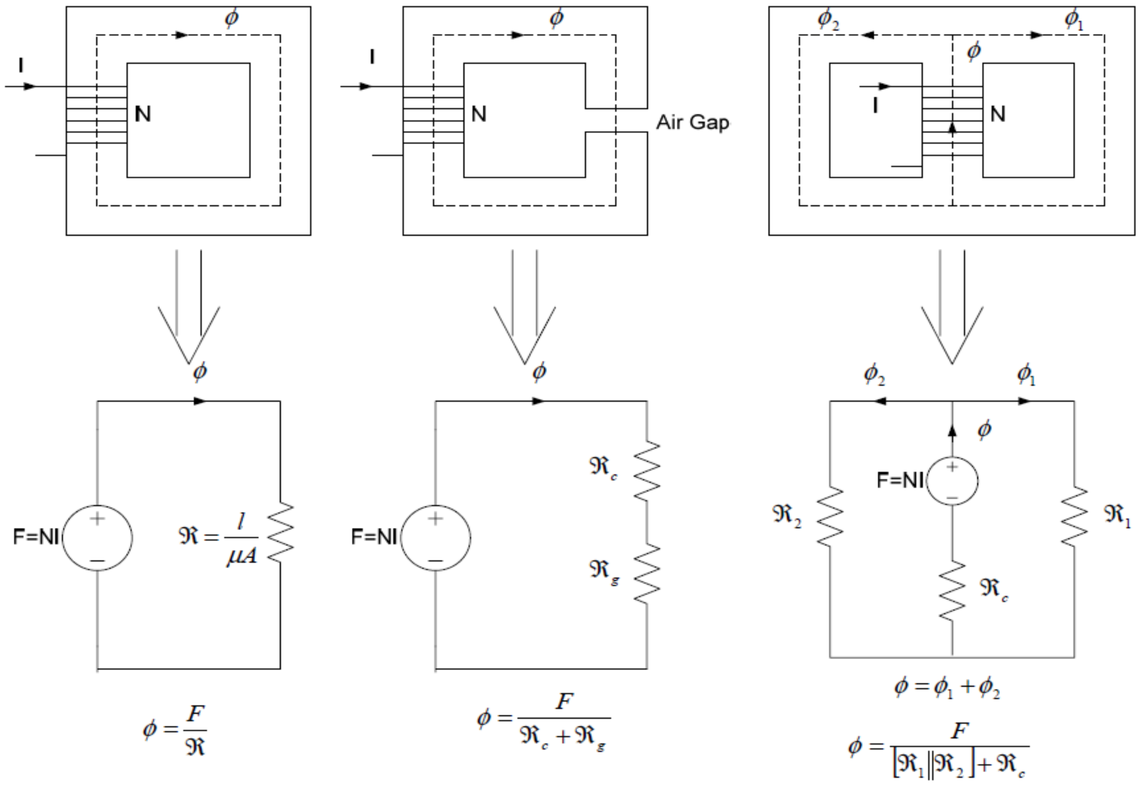
$$\Re = \frac{l_c}{\mu A}$$

| ***ELECTRIC CIRCUIT*** | ***MAGNETIC CIRCUIT*** |
|---|---|
| E = EMF | F = MMF |
| R = Resistance | $\Re$ = Reluctance |
| I = Current | $\phi$ = Flux |
| σ = Conductivity | μ = Permeability |
| $E = RI$ | $mmf = \Re\phi$ |
| $R = \frac{l}{\sigma A} = \frac{\rho l}{A}$ | $\Re = \frac{l}{\mu A}$ |
| $R_{series} = R_1 + R_2 + .... + R_N$ | $\Re_{series} = \Re_1 + \Re_2 + ..... + \Re_N$ |
| $1/R_{parallel} = 1/R_1 + 1/R_2 + .... + 1/R_N$ | $1/\Re_{parallel} = 1/\Re_1 + 1/\Re_2 + ..... + 1/\Re_N$ |

In order to analyze any magnetic circuit, two steps are mandatory as illustrated by figure given below.

**Step #1:** Find the electric equivalent circuit that represents the magnetic circuit.

**Step #2:** Analyze the electric circuit to solve for the magnetic circuit quantities.

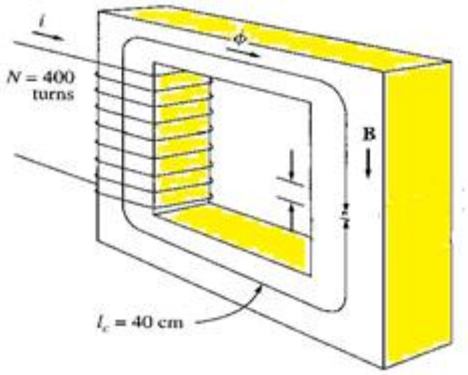


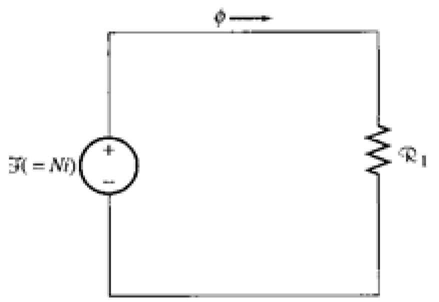

**<u>Example (1):</u>**

A ferromagnetic core with mean path length is 40cm. The CSA of the core is 12cm$^2$, the relative permeability of the core is 4000, and the coil of wire on the core has 400 turns. Find:

(a) The reluctance of the flux path,

(b) The current required to produce a flux density of 0.5T in the core.




The analogous circuit of the core is represented as shown above where reluctance $R_1$ represents the core

$$R_1 = \frac{40 \times 10^{-2}}{4000 \times 4\pi \times 10^{-7} \times 12 \times 10^{-4}} = 66314.5596\ At/Wb$$

$\Phi = B \times A = 0.5 \times 12{\times}10^{-4} = 6{\times}10^{-4}$ Wb

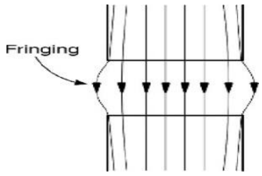
$N{\times}I = \Phi \times R_{total} = 6{\times}10^{-4} \times 66314.5596 = 39.7887$ At

I = 39.7887 / 400 = 99.4718 mA

The fringing effect results from the presence of the air gap in the magnetic circuit. The main consequence of the fringing effect is to make the magnetic flux density of the air gap ($B_g$) different from the flux density of the core ($B_c$) due to the path of the flux.

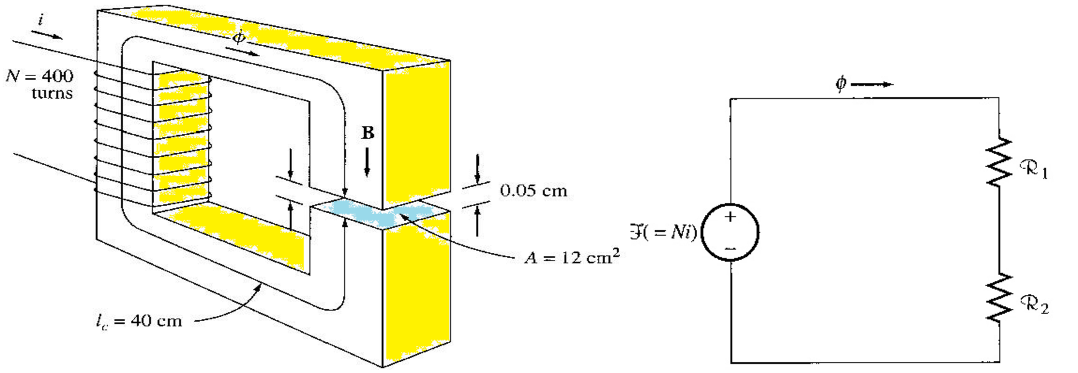
$$\phi_c = \phi_g \,,$$

but $$B_c \neq B_g$$




**Example (2)**

A ferromagnetic core, given in Example (1), there is a small gap of 0.05cm in the structure of the otherwise whole core. Assume that fringing effect is neglected. Find:
(a) The total reluctance of the flux path (iron plus air gap)
(b) The current required to produce a flux density of 0.5T in the air gap.




The analogous circuit of the core is represented as shown above where reluctance $R_1$ represents the core and $R_2$ represent the air gap.

$$R_1 = \frac{40 \times 10^{-2}}{4000 \times 4\pi \times 10^{-7} \times 12 \times 10^{-4}} = 66314.5596 \quad At/Wb$$

$$R_2 = \frac{0.05 \times 10^{-2}}{1 \times 4\pi \times 10^{-7} \times 12 \times 10^{-4}} = 331572.7981 \quad At/Wb$$

The two reluctances are connected in series

$R_{total} = R_1 + R_2 = 397887.3577$ At/Wb ##

$\Phi = B \times A_g = 0.5 \times 12\times10^{-4} = 6\times10^{-4}$ Wb

$N\times I = \Phi \times R_{total} = 6\times10^{-4} \times 397887.3577 = 238.7324$ At

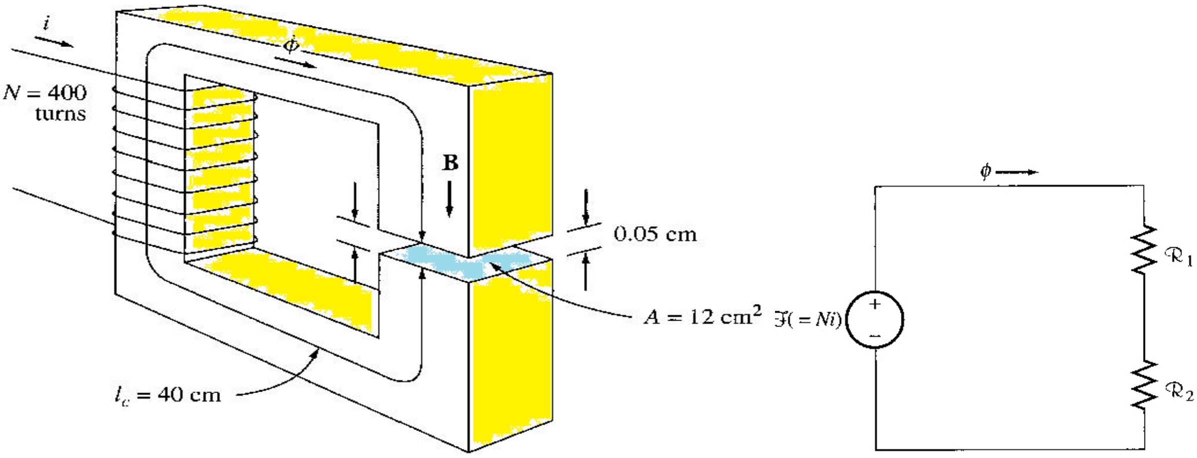
I = 238.7324 / 400 = 0.5968 A

**Example (3)**

A ferromagnetic core given in Example (2). Assume that fringing effect in the air gap increases the effective CSA of the gap by 5%. Find:

(a) The total reluctance of the flux path (iron plus air gap)

(b) The current required to produce a flux density of 0.5T in the air gap.




The analogous circuit of the core is represented as shown above where reluctance $R_1$ represent the core and $R_2$ represent the air gap.

$$R_1 = \frac{40 \times 10^{-2}}{4000 \times 4\pi \times 10^{-7} \times 12 \times 10^{-4}} = 66314.5596 \quad At/Wb$$

$$R_2 = \frac{0.05 \times 10^{-2}}{1 \times 4\pi \times 10^{-7} \times 1.05 \times 12 \times 10^{-4}} = 315783.6172 \quad At/Wb$$

The two reluctances are connected in series

$R_{total} = R_1 + R_2 = 382098.1768$ At/Wb ##

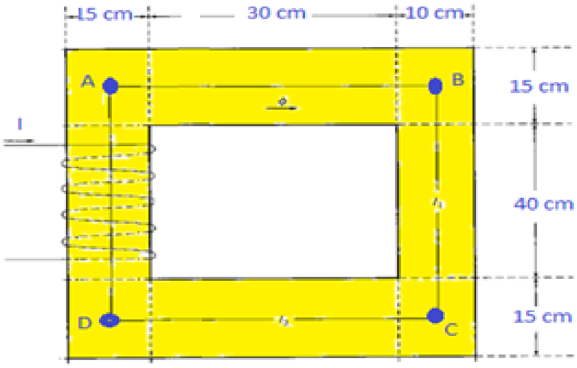
$\Phi = B \times A_g = 0.5 \times 1.05 \times 12\times10^{-4} = 6.3\times10^{-4}$ Wb

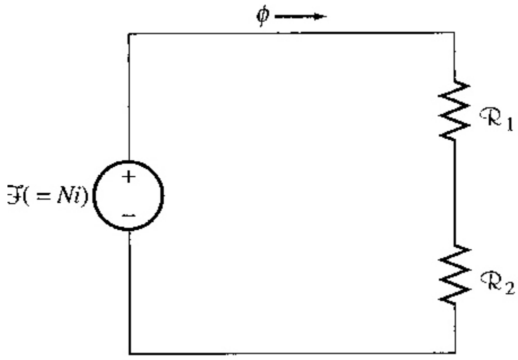
$N\times I = \Phi \times R_{total} = 6.3\times10^{-4} \times 382098.1768 = 240.7219$ At

$I = 240.7219 / 400 = 0.6018$ A

**Example (4):**

A ferromagnetic core is shown. Three sides of this core are of uniform width, while the fourth side is somewhat thinner. The depth of the core (into the page) is 10cm, and the other dimensions are shown in the figure. There is a 200 turn coil wrapped around the left side of the core. Assuming relative permeability $\mu_r$ of 2500, how much flux will be produced by a 1A input current?




Calculate length of each section:

$L_{AB}=L_{CD}$ = 7.5+30+5 = 42.5 cm

$L_{BC}=L_{DA}$ = 7.5+40+7.5 = 55 cm = 0.55 m

$L_{CDAB}$ = 42.5 + 55 + 42.5 = 140 cm = 1.4 m

Calculate the reluctance:

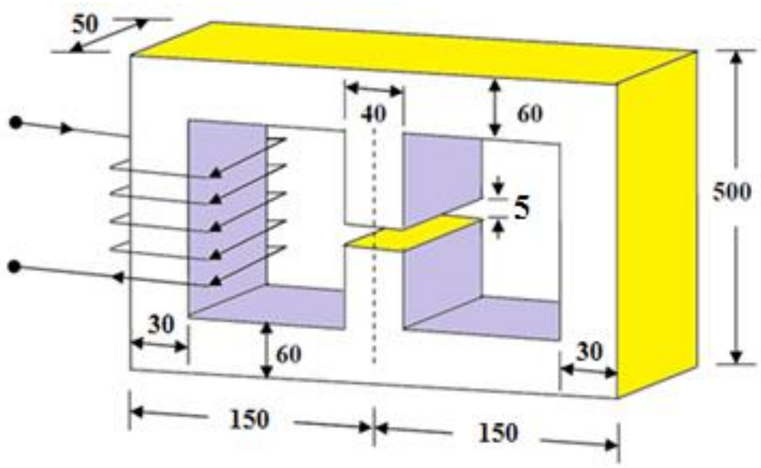
$$\Re_{CDAB} = \frac{l_{CDAB}}{\mu\, A_1} = \frac{1.4}{2500 \times 4\pi \times 10^{-7} \times 15 \times 10 \times 10^{-4}} = 29708.92271$$

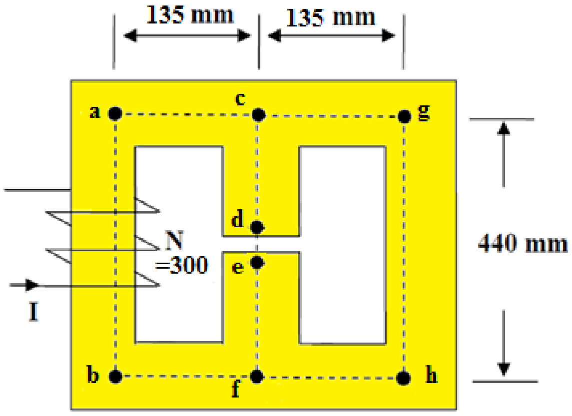
$$\Re_{BC} = \frac{l_{BC}}{\mu\, A_2} = \frac{0.55}{2500 \times 4\pi \times 10^{-7} \times 10 \times 10 \times 10^{-4}} = 17507.04374$$

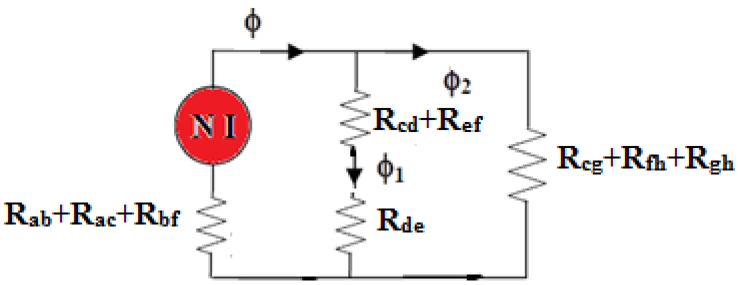
200×1 = $\Phi\times(\Re_{CDAB} + \Re_{BC}) \rightarrow \Phi$ = 4.236 m Wb.

**Example (5):**

In the magnetic circuit shown below with all dimensions in mm, calculate the required current to be passed in the coil having 300 turns in order to establish a flux of 2.28 mWb in the air gap. Consider the fringing effect at the air gap by 7% and the relative permeability of the core is 4000






$$R_{ab} = R_{gh} = \frac{440 \times 10^{-3}}{4000 \times 4\pi \times 10^{-7} \times 30 \times 50 \times 10^{-6}} = 58356.81247$$

$$R_{bf} = R_{ac} = R_{cg} = R_{fh} = \frac{135 \times 10^{-3}}{4000 \times 4\pi \times 10^{-7} \times 60 \times 50 \times 10^{-6}} = 8952.465549$$

$$R_{cd} = R_{ef} = \frac{217.5 \times 10^{-3}}{4000 \times 4\pi \times 10^{-7} \times 40 \times 50 \times 10^{-6}} = 21635.1251$$

$$R_{de} = \frac{5 \times 10^{-3}}{4\pi \times 10^{-7} \times (40 \times 50 \times 10^{-6}) \times 1.07} = 1859286.718$$

$$MMF_1 = \varphi_1(R_{ef} + R_{cd} + R_{de}) = 2.28 \times 10^{-3} \times 1902556.968 = 4337.823$$

$$MMF_2 = \varphi_2(R_{cg} + R_{fh} + R_{gh}) = 4337.823$$

$$\varphi_2 = 56.881 \times 10^{-3}\ Wb$$

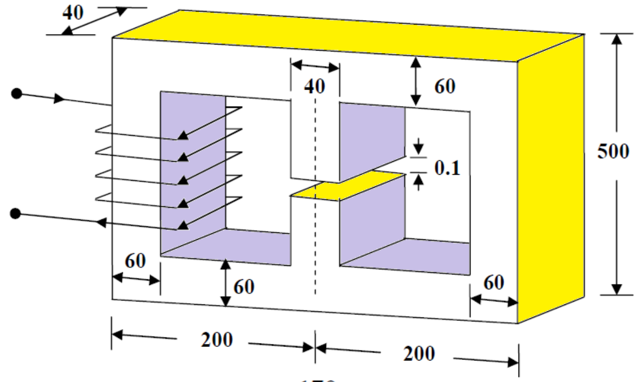
$$\varphi = \varphi_1 + \varphi_2 = (2.28 + 56.881) \times 10^{-3} = 59.161 \times 10^{-3}\ Wb$$

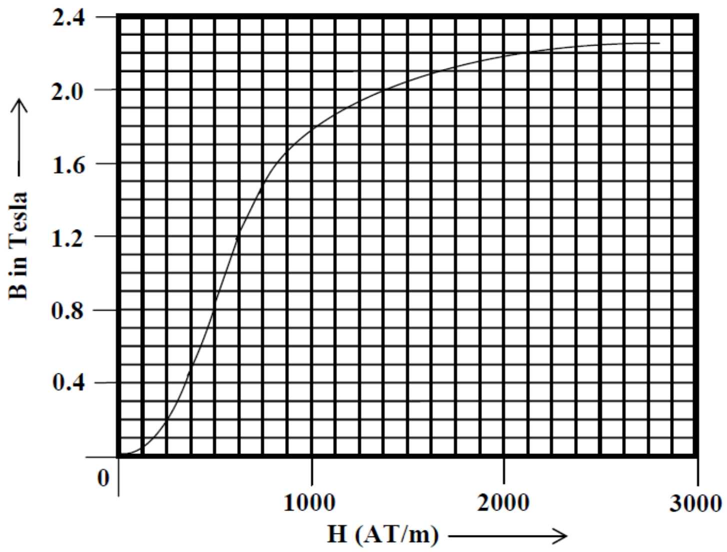
$$N \times i = \varphi(R_{ab} + R_{ac} + R_{bf}) + MMF_1 = 59.161 \times 10^{-3} \times 76261.74357 + 4337.823$$

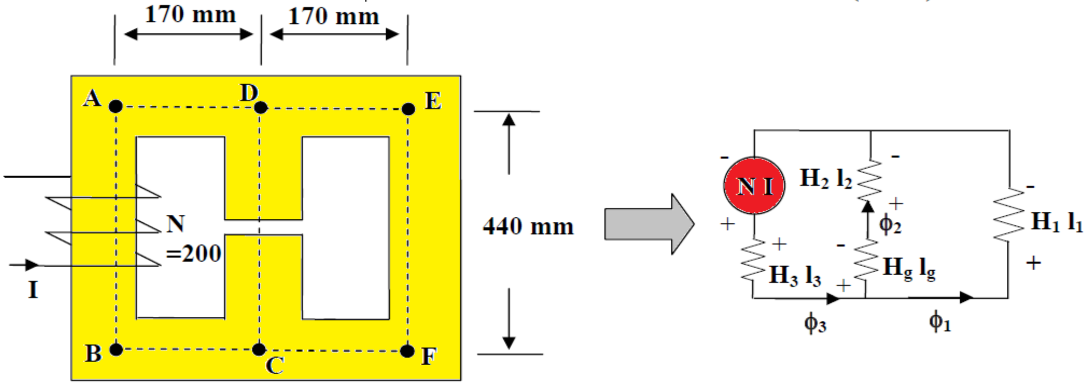
$$i = \frac{8849.544}{300} = 29.4985\ A$$

**Example (6):**

In the magnetic circuit shown in Figure below with all dimensions in mm, calculate the required current to be passed in the coil having 200 turns in order to establish a flux of 1.28 mWb in the air gap. Neglect fringing effect and leakage flux. The B-H curve of the material is given. Permeability of air may be taken as, $\mu_0 = 4\pi \times 10^{-7}$ H/m




The airgap flux $\phi_g$ is $\phi_2 = 1.28\times10^{-3}$ Wb

The cross-sectional area of central limb = $40\times10^{-3}\times40\times10^{-3} = 16\times10^{-4}$ m$^2$

Flux density at airgap $B_g = B_2 = 1.28\times10^{-3} \div 16\times10^{-4} = 0.8$ Tesla

Flux intensity at airgap $H_g = B_g \div \mu_0 = 0.8 \div 4\pi\times10^{-7} = 636619.7724$ AT/m

MMF required for airgap $H_g\,l_g = 636619.7724\times0.1\times10^{-3} = 63.662$ AT

Now we must calculate the mmf required in the iron portion of the central limb as follows:

| | | |
|---|---|---|
| flux density, $B_2$ | = | 0.8 T ∵ fringing & leakage neglected |
| corresponding H from graph, $H_2$ | ≈ | 500 AT /m |
| Mean iron length, $l_2$ | = | (440 – 0.1) mm ≈ 0.44 m |
| mmf required for iron portion, $H_2 l_2$ | = | 220 AT |
| Total mmf required for iron & air gap, | = | (220 + 63.66) AT |
| $mmf_{CD}$ | = | 283.66 AT. |

Due to parallel connection, mmf acting across path 1 is same as mmf acting across path 2. Our intention here, will be to calculate $\phi_1$ in path 1.

$$\text{mean length of the path, } l_1 = l_{DE} + l_{EF} + l_{FC} = 2 \times 170 + 440 \text{ mm} = 0.78 \text{ m}$$

$$\because H_1 = \frac{283.66}{0.78} = 363.67 \text{ AT/m}$$

$$\text{corresponding flux density from graph, } B_1 \approx 0.39 \text{ T}$$

$$\therefore \text{flux, } \phi_1 = B_1 A_1 = 0.39 \times 24 \times 10^{-4} = 0.94 \times 10^{-3} \text{ Wb}$$

we calculate the mmf necessary to drive $\phi_3$ in path 3 as follows.

$$\text{flux in path 3, } \phi_3 = \phi_1 + \phi_2 = 2.22 \times 10^{-3} \text{ Wb}$$

$$\text{flux density, } B_3 = \frac{\phi_3}{A_3} = \frac{2.22 \times 10^{-3}}{24 \times 10^{-4}}$$

$$\therefore B_3 = 0.925 \text{ T}$$

$$\text{corresponding H from graph, } H_3 \approx 562.5 \text{ AT/m}$$

$$\text{mean length of path 3, } l_3 = 2 \times 170 + 440 \text{ mm} = 0.78 \text{ m}$$

$$\text{total mmf required for path 3} = H_3 l_3 = 562.5 \times 0.78 = 438.7 \text{ AT}$$

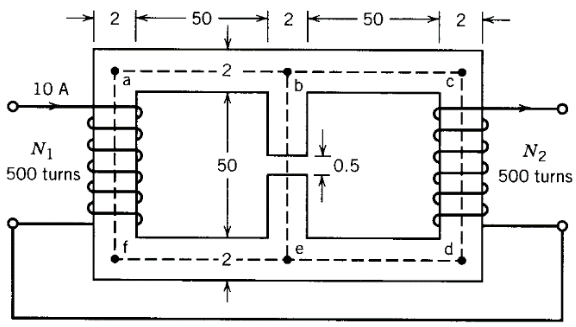
$$\therefore \text{mmf to be supplied by the coil, } NI = 283.66 + 438.7 \text{ AT}$$

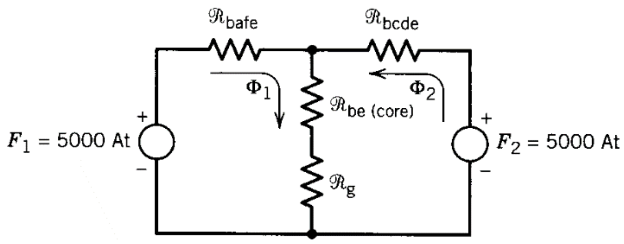
$$\text{or } 200I = 722.36 \text{ AT}$$

$$\therefore \text{exciting current needed, } I = \frac{722.36}{200} \text{ A} = 3.61 \text{ A}$$

**Example (7):**

For the magnetic circuit shown below, the relative permeability of the ferromagnetic material of the core is 1200. Neglect leakage and fringing. All dimensions are in cm, and the magnetic material has a square CSA. Determine the airgap flux and the magnetic field intensity in the airgap.






$F_1 = N_1 I_1 = 500 \times 10 = 5000$ At $\qquad F_2 = N_2 I_2 = 500 \times 10 = 5000$ At

$\mu_c = 1200\mu_0 = 1200 \times 4\pi 10^{-7}$

$$\mathcal{R}_{bafe} = \frac{l_{bafe}}{\mu_c A_c} = \frac{3 \times 52 \times 10^{-2}}{1200 \times 4\pi 10^{-7} \times 4 \times 10^{-4}} = 2.58 \times 10^6 \text{ At/Wb}$$

From symmetry $\quad \mathcal{R}_{bcde} = \mathcal{R}_{bafe}$

$$\mathcal{R}_g = \frac{l_g}{\mu_0 A_g} = \frac{5 \times 10^{-3}}{4\pi 10^{-7} \times 2 \times 2 \times 10^{-4}} = 9.94 \times 10^6 \text{ At/Wb}$$

$$\mathcal{R}_{be(core)} = \frac{l_{be(core)}}{\mu_c A_c} = \frac{51.5 \times 10^{-2}}{1200 \times 4\pi 10^{-7} \times 4 \times 10^{-4}} = 0.82 \times 10^6 \text{ At/Wb}$$

The loop equations are

$$\Phi_1(\mathcal{R}_{bafe} + \mathcal{R}_{be} + \mathcal{R}_g) + \Phi_2(\mathcal{R}_{be} + \mathcal{R}_g) = F_1$$

$$\Phi_1(\mathcal{R}_{be} + \mathcal{R}_g) + \Phi_2(\mathcal{R}_{bcde} + \mathcal{R}_{be} + \mathcal{R}_g) = F_2$$

or

$$\Phi_1(13.34 \times 10^6) + \Phi_2(10.76 \times 10^6) = 5000$$

$$\Phi_1(10.76 \times 10^6) + \Phi_2(13.34 \times 10^6) = 5000$$

or

$$\Phi_1 = \Phi_2 = 2.067 \times 10^{-4} \text{ Wb}$$

The air gap flux is

$$\Phi_g = \Phi_1 + \Phi_2 = 4.134 \times 10^{-4} \text{ Wb}$$

The air gap flux density is

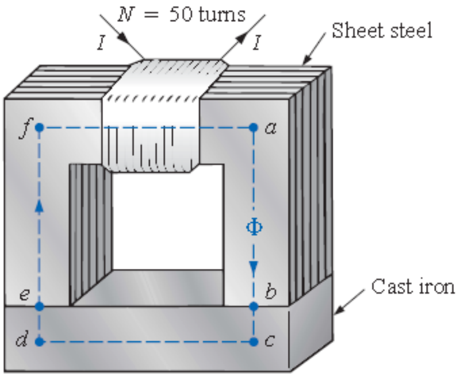
$$B_g = \frac{\Phi_g}{A_g} = \frac{4.134 \times 10^{-4}}{4 \times 10^{-4}} = 1.034 \text{ T}$$

The magnetic intensity in the air gap is

$$H_g = \frac{B_g}{\mu_0} = \frac{1.034}{4\pi 10^{-7}} = 0.822 \times 10^6 \text{ At/m}$$

**Example (8):**

The electromagnet shown in figure below has picked up a section of cast iron. Determine the current I required to establish the indicated flux in the core assuming that the relative permeability of sheet steel is 6165 and for cast iron is 2662.




$l_{ab} = l_{cd} = l_{ef} = l_{fa} = 4$ in.
$l_{bc} = l_{de} = 0.5$ in.
Area (throughout) = 1 in.$^2$
$\Phi = 3.5 \times 10^{-4}$ Wb

First, we must first convert to the metric system. However, since the area is the same throughout, we can determine the length for each material rather than work with the individual sections:

$l_{efab}$ = 4 in. + 4 in. + 4 in. = 12 in. = 12×0.0254 = 0.3048 m

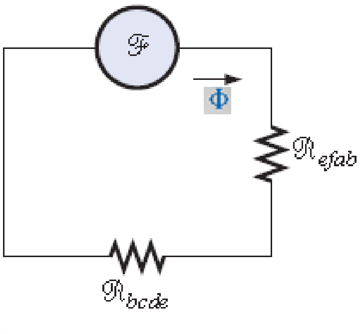
$l_{bcde}$ = 0.5 in. + 4 in. + 0.5 in. = 5 in × 0.0254 = 0.127 m

The C.S.A = 1× $(0.0254)^2$ = $6.452\times10^{-4}$ m$^2$

Now we need to calculate the reluctance for each section:

$$\Re_{efab} = \frac{l_{efab}}{\mu_{steel}\, A} = \frac{0.3048}{6165 \times 4\pi \times 10^{-7} \times 6.452 \times 10^{-4}} = 60978.62945$$

$$\Re_{bcde} = \frac{l_{bcde}}{\mu_{iron}\, A} = \frac{0.127}{2662 \times 4\pi \times 10^{-4} \times 6.452 \times 10^{-4}} = 58842.54486$$




$50\times I = \Phi\times(\Re_{efab} + \Re_{bcde}) \rightarrow I = 0.839$ A

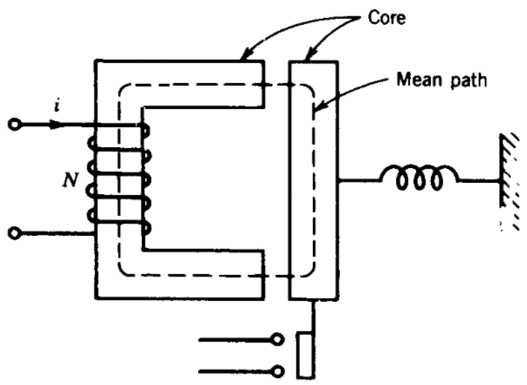
**Example (9):**

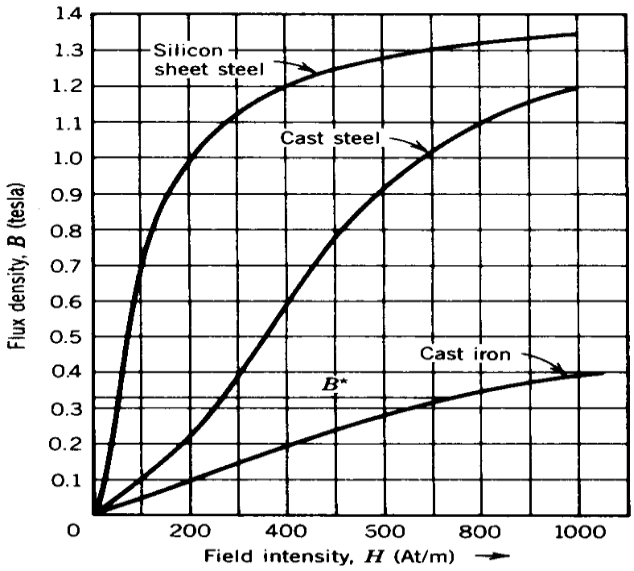
For the cast-steel magnetic circuit relay with N = 500 and the mean path is 360 mm and the air gap lengths are 1.5 mm each. A flux density of 0.8 T is required to actiuate the relay. Using the B-H curve of the cast steel core shown below, find:

a) The current in the coil,

b) The relative permeability of the core,

c) If the air gaps become zero, calculate the required current in the coil.




$$B_c = 0.8\text{ T}, \qquad H_c = 510\text{ At/m}$$

$$\text{mmf } F_c = H_c l_c = 510 \times 0.36 = 184\text{ At}$$

For the air gap,

$$\text{mmf } F_g = H_g 2l_g = \frac{B_g}{\mu_0} 2l_g = \frac{0.8}{4\pi 10^{-7}} \times 2 \times 1.5 \times 10^{-3}$$

$$= 1910\text{ At}$$

Total mmf required:

$$F = F_c + F_g = 184 + 1910 = 2094\text{ At}$$

Current required:

$$i = \frac{F}{N} = \frac{2094}{500} = 4.19\text{ amps}$$

**(b)** Permeability of core:

$$\mu_c = \frac{B_c}{H_c} = \frac{0.8}{510} = 1.57 \times 10^{-3}$$

Relative permeability of core:

$$\mu_r = \frac{\mu_c}{\mu_0} = \frac{1.57 \times 10^{-3}}{4\pi 10^{-7}} = 1250$$

**(c)**

$$F = H_c l_c = 510 \times 0.36 = 184\text{ At}$$

$$i = \frac{184}{500} = 0.368\text{ A}$$

Note that if the air gap is not present, a much smaller current is required to establish the same flux density in the magnetic circuit.

**Example (10):**

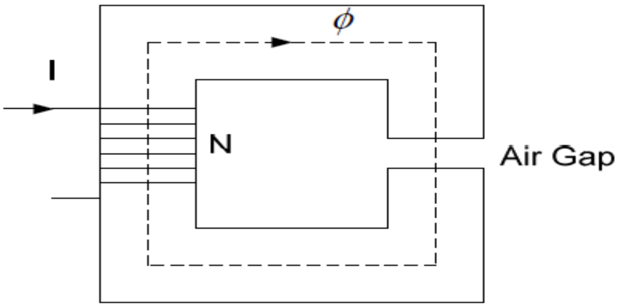
The magnetic circuit shown below has the following dimensions: $A_c$= 16 cm$^2$, $l$ = 40cm, $l_g$ = 0.5mm and $N$ = 350 turns. The core is made of a material with the $B$-$H$ relationship given below. For $B$ = 1.0 T in the core, find:

a) The flux φ and the total flux linkage λ, where λ=$N$ φ.

b) The required current to set this flux if there is no air gap.

c) The required current with the presence of an air gap.

| B (Tesla) | H (A.T) |
|---|---|
| 0.6 | 12.5 |
| 0.8 | 15.0 |
| 1.0 | 20.0 |
| 1.2 | 31.0 |
| 1.4 | 55.0 |




a) $$\phi = BA_c = 1.0\times16\times10^{-4} = 1.6\,mWb$$

$$\lambda = N\phi = 350\times1.6\times10^{-3} = 0.56\,Wb.t$$

b) With no air-gap

$$F = \Re_c\phi = NI$$

$$\therefore I = \frac{\Re_c\phi}{N}$$

$$\Re_c = \frac{l}{\mu_c A_c},$$

$$\mu_c = \frac{B}{H} = \frac{1.0}{20.0} = 0.05$$

$$\Re_c = \frac{40\times10^{-2}}{0.05\times16\times10^{-4}} = 5000\ At/wb$$

$$\therefore I = \frac{5000\times1.6\times10^{-3}}{350} = 22.86\ mA$$

c) With air-gap

$$F = NI = (\Re_c + \Re_g)\phi$$

$$\Re_c = \frac{l_c - l_g}{\mu_c A_c} \cong 5000,$$

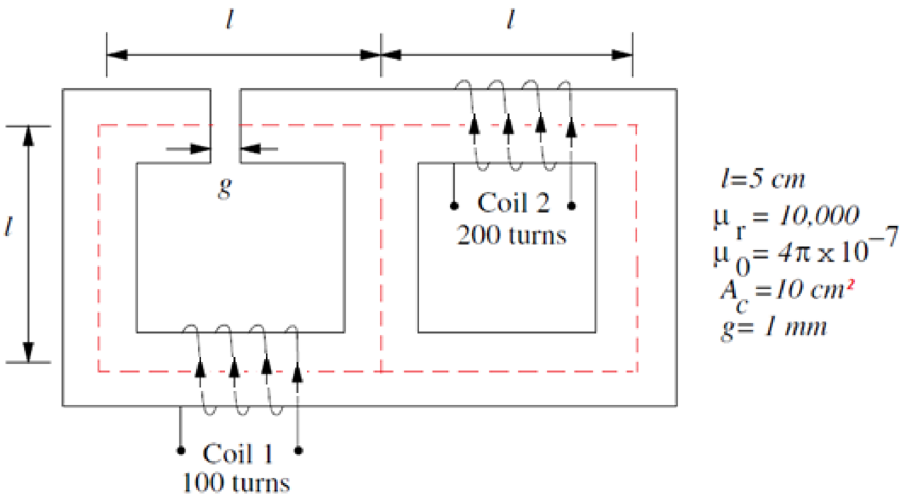
$$\Re_g = \frac{l_g}{\mu_g A_g} = \frac{0.5\times10^{-3}}{(4\pi\times10^{-7})\times16\times10^{-4}} = 248{,}679.6$$

$$I = \frac{(\Re_c + \Re_g)\phi}{N} = 1.16\ A$$

In this example, the current needed to set the same flux in case of magnetic circuits with air gap compared to those circuits without air-gap is much higher.

**Example (11):**

Two coils are wound on a magnetic core with an air-gap as shown in figure below. Find all of the magnetic fluxes in this magnetic system, assuming that the applied electric currents $i_1$ = 4A and $i_2$ = 2A.




It is convenient to think of the flux contour as consisting of several parts of different reluctances. Let $R_1$ denote the lump reluctance associated with parts (a) and (b) of the magnetic circuit. The length of the contour representing this part of the magnetic system is $l_1 = l + l + (l - g) = 3l - g = 14.9$ cm

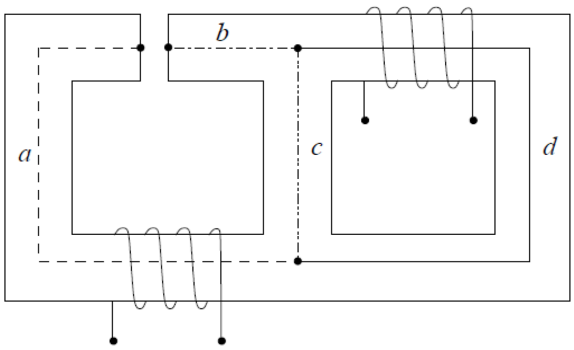
$$\mathcal{R}_1 = \frac{3l - g}{\mu A_c}$$

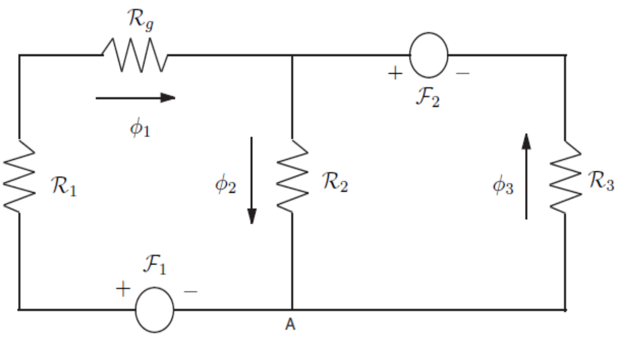
Similarily, we calculate the reluctances $R_2$, $R_3$ of parts (c) and (d) of the magnetic circuit as well as the reluctance $R_g$ of the air gap:

$$\mathcal{R}_2 = \frac{l}{\mu A_c}, \quad \mathcal{R}_3 = \frac{3l}{\mu A_c}, \quad \mathcal{R}_g = \frac{g}{\mu_0 A_c}$$

$$\mathcal{R}_1 = 1.1857 \times 10^4,$$
$$\mathcal{R}_2 = 3.9789 \times 10^3,$$
$$\mathcal{R}_3 = 1.1937 \times 10^4,$$
$$\mathcal{R}_g = 7.9577 \times 10^5,$$

$F_1 = 100 \times 4 = 400$ At and $F_2 = 200 \times 2 = 400$ At




Applying KVL for the loop $AF_1R_1R_gR_2A$:

$F_1 = (R_1+R_g)\,\varphi_1 + R_2\,\varphi_2$

Applying KVL for the loop $A\,R_2F_2R_3A$:

$F_2 = R_3\,\varphi_3 + R_2\,\varphi_2$

Applying KCL at node A: $\varphi_3 + \varphi_1 = \varphi_2$

Substituting in the above two equations:

$F_1 = (R_1+R_g)\,\varphi_1 + R_2\,(\varphi_3 + \varphi_1) = (R_1+R_g+R_2)\,\varphi_1 + R_2\,\varphi_3$

$400 = 811605.9\,\varphi_1 + 3978.9\,\varphi_3$ -----(1)

$F_2 = R_3\,\varphi_3 + R_2\,(\varphi_3 + \varphi_1) = R_2\,\varphi_1 + (R_2+R_3)\,\varphi_3$

$400 = 3978.9\,\varphi_1 + 15915.9\,\varphi_3$ -----(2)

Solving the above two equations:

$0 = 807627\ \varphi_1 - 11937\ \varphi_3$

$\varphi_1 = 0.01478\ \varphi_3$ -----(3)

Substituting with this value in (2)

$400 = 3978.9\ (0.01478\ \varphi_3) + 15915.9\ \varphi_3 = 15974.7081\ \varphi_3$

Then $\varphi_3 = 25.0396$ mWb

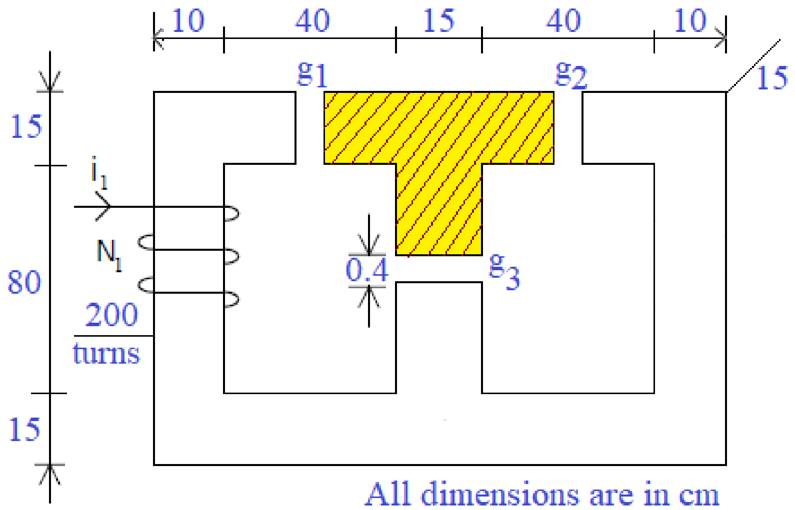
Substituting with the value of $\varphi_3$ in (3)

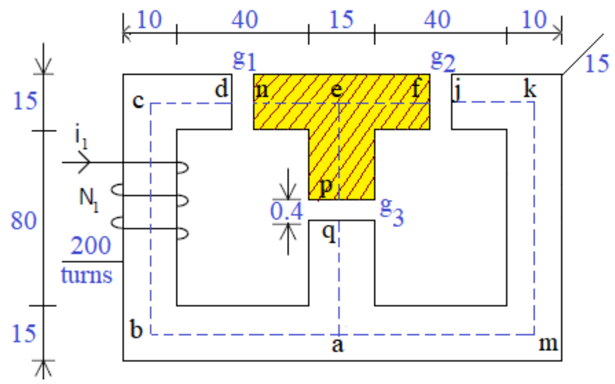
Then $\varphi_1 = 0.37$ mWb

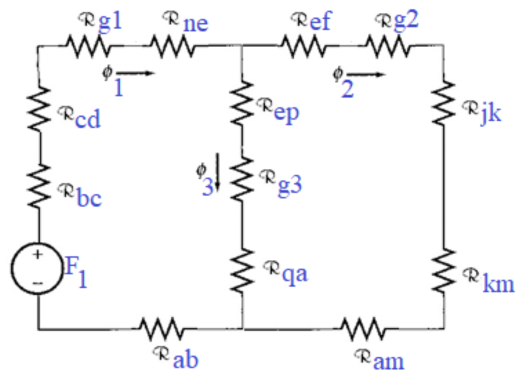
Since $\varphi_2 = \varphi_1 + \varphi_3 = 25.4096$ mWb

**Example (12):**

The magnetic circuit shown below has three identical air gaps ($g_1$, $g_2$ and $g_3$) and one coil with 200 turns. The circuit is made of two different materials (shaded material has a relative permeability of 1800 but the clear one has a relative permeability of 3000). Calculate the current ($i_1$) to be passed in the coil ($N_1$) to establish a flux density of 0.6 T along the air gap ($g_3$). Consider the fringing effect at the air gap ($g_3$) by 7% and the fringing effect is ignored at the other air gaps.






Reluctance Calculation:

$$\Re_{AB} = \Re_{AM} = \frac{l}{\mu\, A} = \frac{52.5 \times 10^{-2}}{3000 \times 4\pi \times 10^{-7} \times 15 \times 15 \times 10^{-4}} = 6189.3589\ A.t/Wb$$

$$\Re_{BC} = \Re_{KM} = \frac{l}{\mu\, A} = \frac{95 \times 10^{-2}}{3000 \times 4\pi \times 10^{-7} \times 10 \times 15 \times 10^{-4}} = 16799.6884\ A.t/Wb$$

$$\Re_{CD} = \Re_{JK} = \frac{l}{\mu\, A} = \frac{24.8 \times 10^{-2}}{3000 \times 4\pi \times 10^{-7} \times 15 \times 15 \times 10^{-4}} = 2923.7353\ A.t/Wb$$

$$\Re_{NE} = \Re_{EF} = \frac{l}{\mu\, A} = \frac{27.3 \times 10^{-2}}{1800 \times 4\pi \times 10^{-7} \times 15 \times 15 \times 10^{-4}} = 5364.11105\ A.t/Wb$$

$$\Re_{EP} = \frac{l}{\mu\, A} = \frac{47.3 \times 10^{-2}}{1800 \times 4\pi \times 10^{-7} \times 15 \times 15 \times 10^{-4}} = 9293.8627\ A.t/Wb$$

$$\Re_{QA} = \frac{l}{\mu\, A} = \frac{47.3 \times 10^{-2}}{3000 \times 4\pi \times 10^{-7} \times 15 \times 15 \times 10^{-4}} = 5576.3176\ A.t/Wb$$

The reluctance of air gaps

$$\Re_{G1} = \Re_{G2} = \frac{l_g}{\mu_o\, A} = \frac{0.4 \times 10^{-2}}{4\pi \times 10^{-7} \times 15 \times 15 \times 10^{-4}} = 141471.0605\ A.t/Wb$$

$$\Re_{G3} = \frac{l_{g3}}{\mu_o\, A} = \frac{0.4 \times 10^{-2}}{4\pi \times 10^{-7} \times 1.07 \times 15 \times 15 \times 10^{-4}} = 132215.9444\ A.t/Wb$$

Since the flux density along the air gap $G_3$ = 0.6 T, then

$$\varphi_3 = B \times A = 0.6 \times (15 \times 15 \times 10^{-4} \times 1.07) = 0.014445\ Wb$$

The MMF through the central leg ($F_3$)

$$F_3 = \varphi_3 \times \left(\Re_{EP} + \Re_{G3} + \Re_{QA}\right) = 0.014445 \times 147086.1247 = 2124.6591\ A.t$$

The MMF through the Right-Hand leg ($F_2$)

$$F_2 = \varphi_2 \times \left(\Re_{EF} + \Re_{G2} + \Re_{JK} + \Re_{KM} + \Re_{AM}\right) = F_3 = 2124.6591\ A.t$$

$$\varphi_2 \times (172747.9542) = 2124.6591\ A.t$$

$$\varphi_2 = 0.0123\ Wb$$

But $\Phi_1 = \Phi_2 + \Phi_3$

$$\varphi_1 = 0.014445 + 0.0123 = 0.026744\ Wb$$

The MMF through the Left-Hand leg ($F_1$)

$$F_1 = 200 \times I_1 = \varphi_1 \times (\Re_{AB} + \Re_{BC} + \Re_{CD} + \Re_{G1} + \Re_{NE}) + F_3$$

$$200 \times I_1 = 0.026744 \times (172747.9542) + 2124.6591 = 6744.6624$$

$$I_1 = 33.7233\ A$$

**Magnetic Behavior of Ferromagnetic Materials:**

Materials which are classified as non-magnetic show a linear relationship between the flux density B and coil current I. In other words, they have constant permeability. Thus, for example, in free space, the permeability is constant. But in iron and other ferromagnetic materials permeability is not constant. For magnetic materials, a much larger value of B is produced in these materials than in free space. Therefore, the permeability of magnetic materials is much higher than $\mu_o$. However, the permeability is not linear anymore but does depend on the current over a wide range. Thus, the permeability is the property of a medium that determines its magnetic characteristics. In other words, the concept of magnetic permeability corresponds to the ability of the material to permit the flow of magnetic flux through it. In electrical machines and electromechanical devices a somewhat linear relationship between B and I is desired, which is normally approached by limiting the current.

Look at the magnetization curve and B-H curve shown in Fig. 15, where the flux density (B) produced in the core is plotted versus the flux intensity (H) producing it. This plot is called a **magnetization curve.** At first, a small increase in H produces a huge increase in B. After a certain point, further increases in H produce relatively smaller increases in B. Finally, there will be no change at all as you increase H further. The region in which the curve flattens out is called saturation region, and the core is said to be **saturated**. The region where B changes rapidly is called **the unsaturated region**. The transition region is called the 'knee' of the curve.

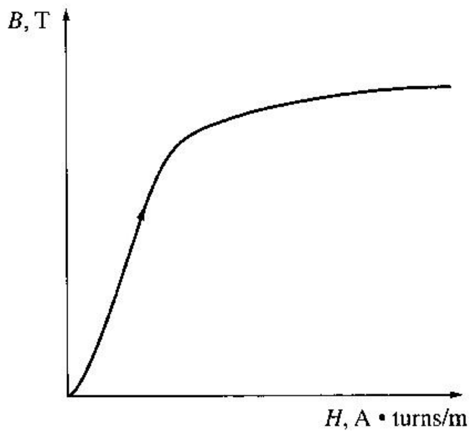



Fig. 15, B-H curve for ferromagnetic materials

Therefore, if the resulting flux has to be proportional to the mmf, then the core must be operated in the unsaturated region. Generators and motors depend on magnetic flux to produce voltage and torque, so they need as much flux as possible. So, they operate near the knee of the magnetization curve (flux not linearly related to the mmf). This non-linearity as a result gives peculiar behaviors to machines. As magnetizing intensity H increased, the relative permeability first increases and then starts to decrease.

**Energy Losses in a Ferromagnetic Core:**

1) **Hysteresis loss**

If an AC current flows through a coil, we expect that during the positive cycle there is a relation between B and H as we discussed in the previous section. On the other hand, during the negative cycle, the B-H relation is a mirror to that obtained in positive cycle. The curve would be as shown in Fig. 16.

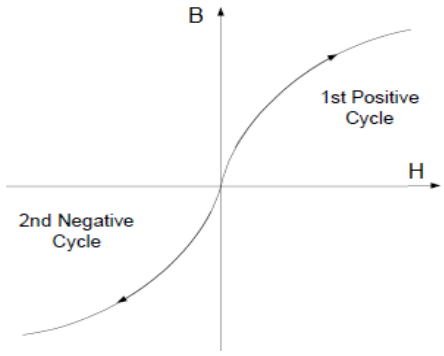
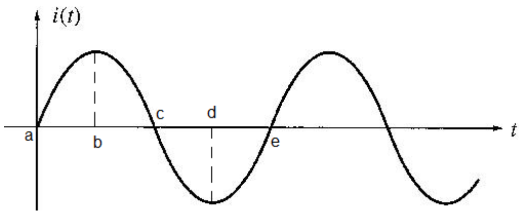



Fig. 16, Theoretical ac magnetic behavior

Unfortunately, the above assumption is only correct provided that the core is 'perfect' i.e. there are no residual flux present during the negative cycle of the ac current flow. A typical flux behavior (or known as hysteresis loop) in a ferromagnetic core is as shown in Fig. 17. HYSTERESIS is the dependence on the preceding flux history and the resulting failure to retrace flux paths.

The explanation to that curve is that, when we apply AC current (assuming flux in the core is initially zero), as current increases, the flux traces the path *ab*. (saturation curve). When the current decreases, the flux traces out a different path from the one when the current increases. When current decreases, the flux traces out path *bcd*. When the current increases again, it traces out path *deb*. From these paths we noted that:

- When MMF is removed, the flux does not go to zero (**residual flux)**. This is how permanent magnets are produced.
- To force the flux to zero, an amount of mmf known as **coercive MMF** must be applied in the opposite direction.



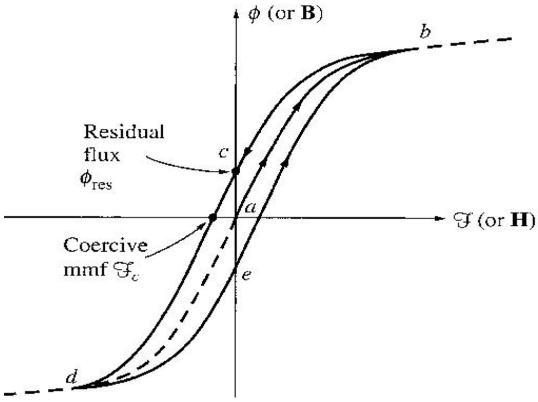



Fig. 17, Hysteresis path

This related to the atoms of iron and similar metals like cobalt, nickel, and some of their alloys tend to have their magnetic fields closely aligned with each other. Within the metal, there is an existence of small regions known as **domains** where in each domain there is a presence of a small magnetic field which randomly aligned through the metal structure such that the net magnetic field is zero, as shown in Fig. 18.

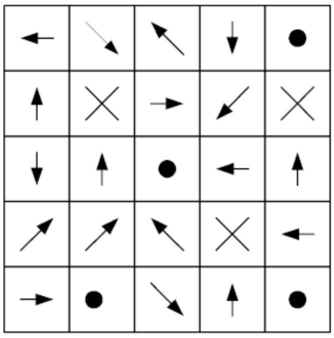


Fig. 18, Magnetic domain orientation in a metal before applying a magnetic field.

When mmf is applied to the core, each magnetic field will align with respect to the direction of the magnetic field. That explains the exponential increase of magnetic flux during the early stage of magnetisation. As more and more domain are aligned to the magnetic field, the total magnetic flux will maintain at a constant level hence as shown in the magnetisation curve (saturation). When mmf is removed, the magnetic field in each domain **will try** to revert to its random state. However, **not all** magnetic field domain's would revert to its random state hence it remained in its previous magnetic field position. This is due to the lack of energy required to disturb the magnetic field alignment.

Hence the material will retain some of its magnetic properties (permanent magnet) up until an external energy is applied to the material. Therefore, in an AC current situation, to realign the magnetic field in each domain during the opposite cycle would require extra mmf (known as coercive mmf). This extra energy requirement is known as **hysteresis loss**. The larger the material, the more energy is required hence the higher the hysteresis loss. Area enclosed in the hysteresis loop formed by applying an AC current to the core is directly proportional to the energy lost in a given ac cycle.

**2) Eddy-current loss**

At first, the changing flux induces voltage within a ferromagnetic core. This voltage cause swirls of current to flow within the core called eddy currents. Energy is dissipated (in the form of heat) because these eddy currents are flowing in a resistive material (iron). The amount of energy lost to eddy currents is proportional to the **size of the paths** they follow within the core. To reduce energy loss, ferromagnetic core should be broken up into small strips, or laminations, and build the core up out of these strips. An insulating oxide or resin is used between the strips, so that the current paths for eddy currents are limited to small areas as shown in Fig. 19.

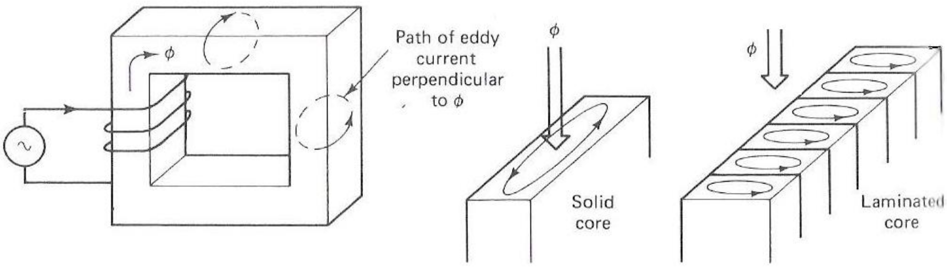



Fig. 19, Eddy-current in ferromagnetic materials

## 2. Synchronous Machine Structures

The synchronous machine (motor or generator) is consists of stator (called armature) and rotor. The stator consists of a cast-iron frame, which supports the armature core, having slots on its inner periphery for housing the armature conductors as shown in Fig. 20.

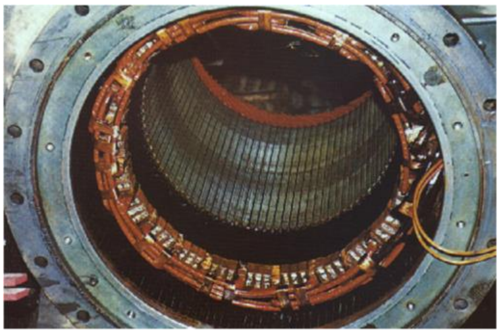
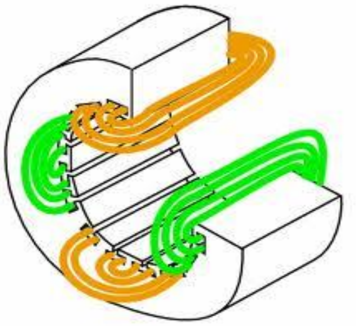



Fig. 20, Stator of a synchronous machine

The armature winding is usually a three phase winding. The rotor is like a flywheel having alternate *N* and *S* poles fixed to its outer edge. The magnetic poles on the rotor may obtain from permanent magnets or from field winding that are excited (or magnetized) from direct current supplied by a DC source at 125 to 600 volts. In most cases, necessary exciting (or magnetizing) current is obtained from a small DC shunt generator which is belted or mounted on the shaft of the alternator itself and called **exciter**. Because the field magnets are rotating, this current is supplied through **two slip rings**. As the exciting voltage is relatively small, the slip-rings and brush gear are of light construction as shown in Fig. 21.

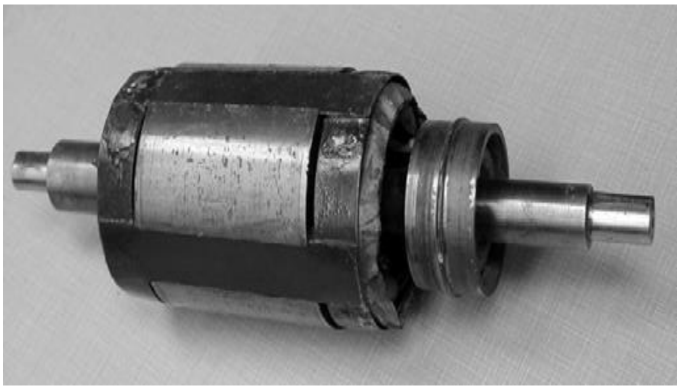
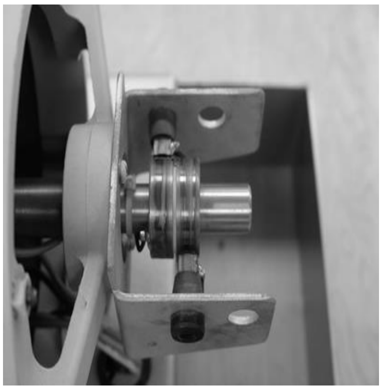



Fig. 21, Slip rings used for the exciter

Figure 22, gives a schematic diagram of the construction of synchronous machines.

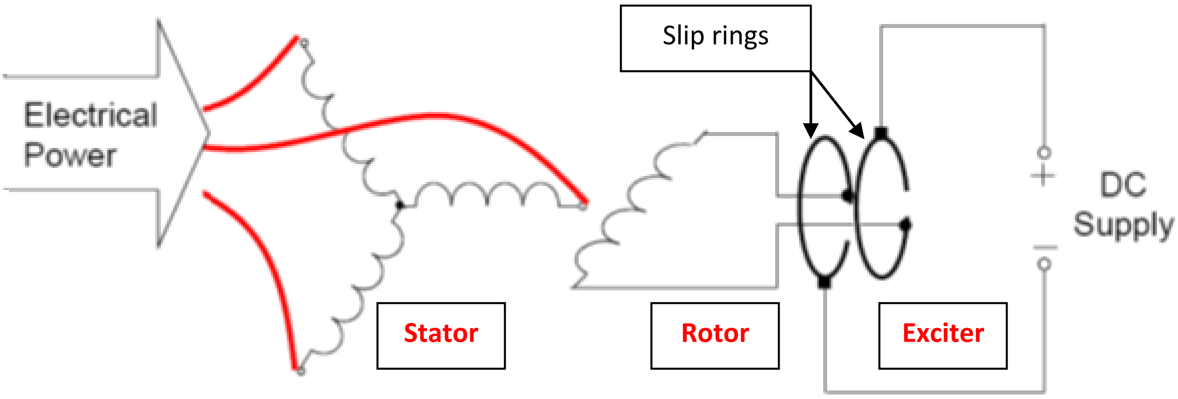



Fig. 22, schematic diagram of the construction of synchronous machines

Recently, brushless excitation systems have been developed in which a 3-phase AC exciter and a group of rectifiers supply DC to the alternator. Hence, brushes, slip-rings and commutator are eliminated as shown in Fig. 23.

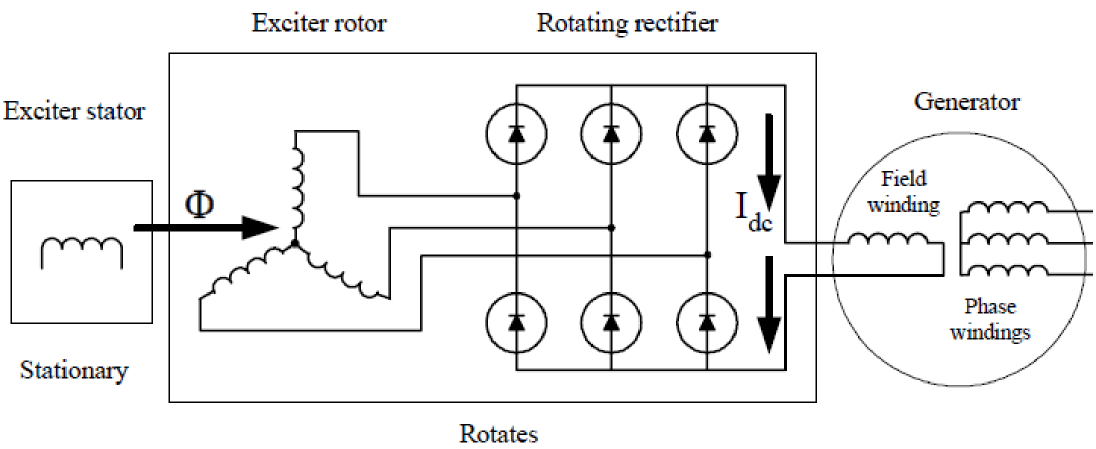



Fig. 23, Brushless exciter

The principal of operation of synchronous machines is illustrated by Fig. 24. A DC current is applied to the rotor winding, which then produces a rotor magnetic field. The rotor is then turned by a prime mover (eg. Steam, water etc.) producing a rotating magnetic field. This rotating magnetic field induces a 3-phase set of voltages within the stator windings of the generator. "Field windings" applies to the windings that produce the main magnetic field in a machine, and "Armature windings" applies to the windings where the main voltage is induced. For synchronous machines, the field windings are on the rotor, so the terms "rotor windings" and "field windings" are used interchangeably.

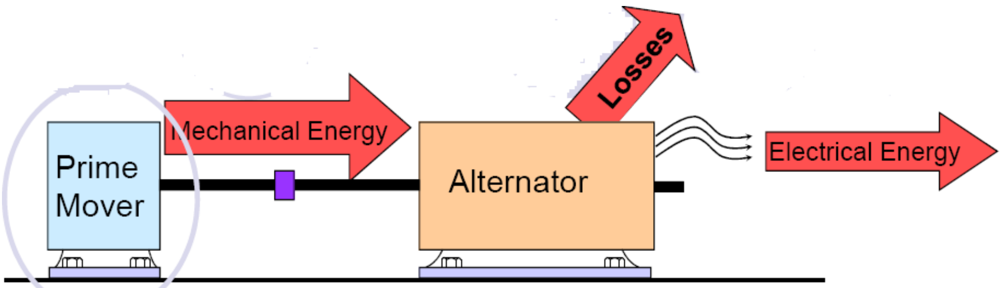
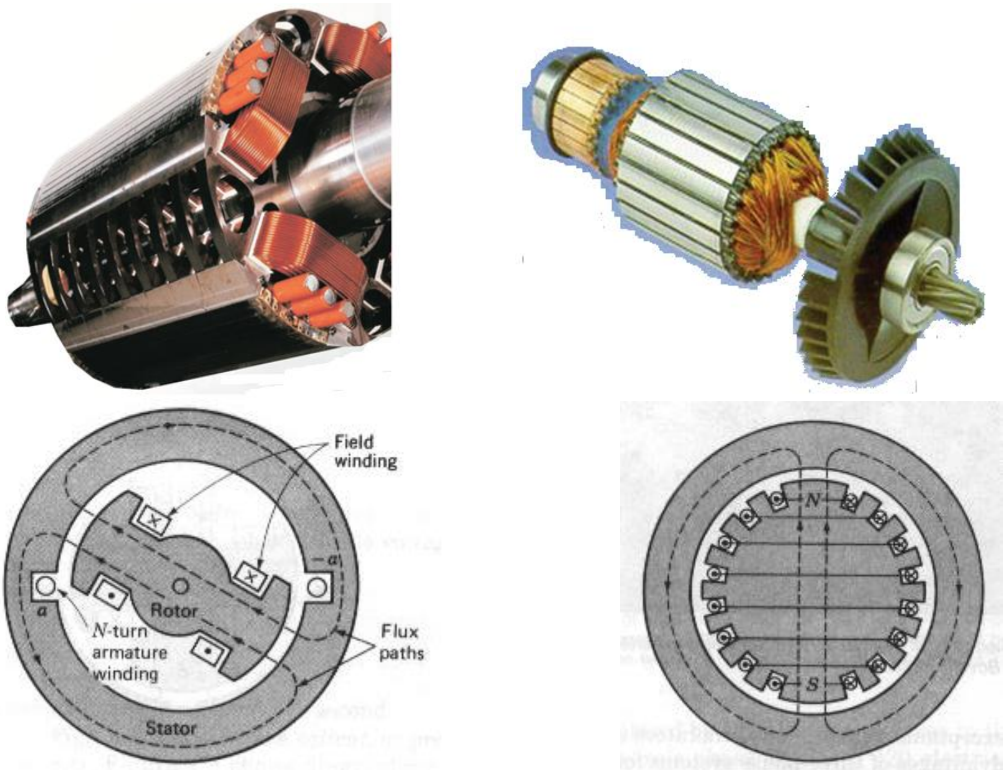



Fig. 24, principal of operation of synchronous machines

There are two types of rotor structures: ***round or cylindrical rotor and salient pole rotor*** as illustrated schematically in Fig. 25. Generally, *cylindrical rotor* structure is used for 2 or 4 poles at most giving *high speed* synchronous machines. The cylindrical rotor has small diameter (1.2m) and long lengths (2 to 5m). Cylindrical rotors are used for high speed turbo generators, like gas and steam turbine generation system. *Salient pole* structure is used for more than 4 poles giving *low speed* applications. The salient rotor has large diameter (2 to 6m) to accommodate several poles and short axial length (1m). Salient pole rotor is used for low speed Hydro generators.




(a) (b)

Fig. 25, Schematic illustration of synchronous machines of
(a) salient rotor and (b) cylindrical rotor structures
(b)

## 2.1 Damper windings

Most of the alternators have their pole-shoes slotted for receiving copper bars of a grid or damper winding (also known as squirrel-cage winding). The copper bars are short-circuited at both ends by heavy copper rings (Fig. 25-a and Fig. 26). These dampers are useful in preventing the hunting (momentary speed fluctuations) in generators and are needed in synchronous motors to provide the starting torque. Turbo-generators usually do not have these damper windings (except in special case to assist in synchronizing) because the solid field-poles themselves act as efficient dampers. It should be clearly understood that under normal running conditions, damper winding does not carry any current because rotor runs at synchronous speed.

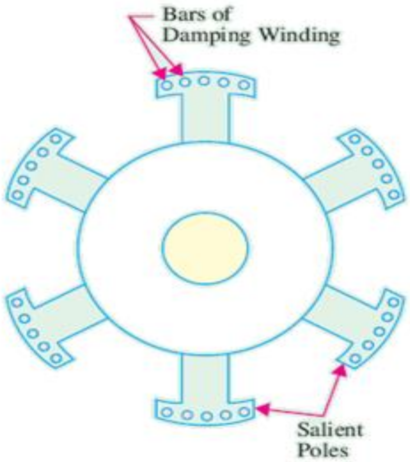



Fig. 26, Damper winding in salient pole machines

## 3. Angle in Electrical and Mechanical Degrees

Consider a synchronous machine with two magnetic poles as shown in Fig. 27-a. The ideal radial distribution of the flux density is sinusoidal along the air gap. When the rotor rotates for one cycle, the induced *emf*, which is also sinusoidal, varies for one cycle as illustrated by the waveforms shown in Fig. 28-a. If we measure the rotor position by **physical or mechanical degrees $\theta_m$** and the phase angles of the flux density and emf by ***electrical degrees $\theta_e$***, in this case, we find the angle measured in mechanical degrees is equal to that measured in electrical degrees, i.e.

$$\theta_e = \theta_m$$

where $\theta_e$ is the angle in electrical degrees and $\theta_m$ the mechanical angle.

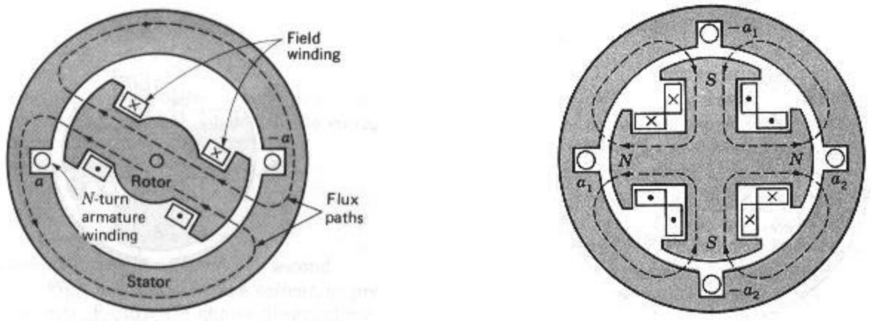



Fig. 27, synchronous machines with

(a) Two poles (b) Four poles

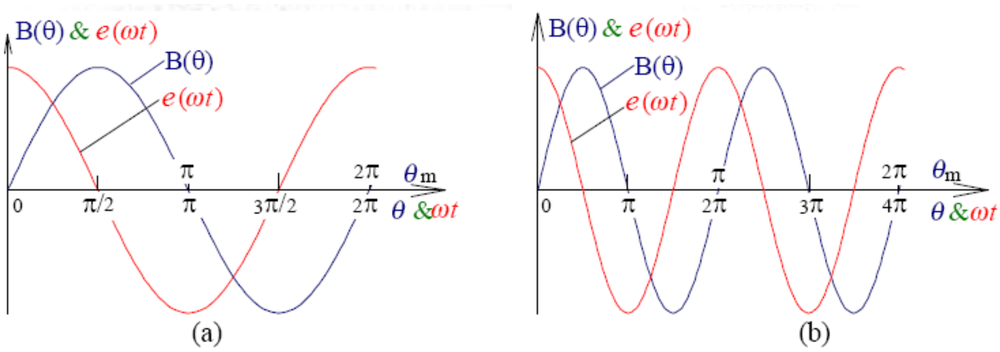



Fig. 28, Flux density distribution in air gap and induced *emf* in armature winding of (a) two pole and (b) four pole synchronous machine

Many synchronous machines have more than two poles. As a specific example, we consider a four pole machine as shown in Fig. 27-b. As the rotor rotates for one cycle ($\theta_m = 2\pi$), the induced *emf* varies for two cycles ($\theta_e = 4\pi$) as shown in Fig. 28-b, and hence

$$\theta_e = 2\,\theta_m$$

For a general case, if a machine has 2*P* poles, where P is the pair poles, the relationship between the electrical and mechanical units of an angle can be deduced as

$$\theta_e = P\,\theta_m$$

Also

$$\omega_e = P\,\omega_m$$

$\omega_e = 2\pi f$ and $\omega_m = 2\pi\, n/60$

$f$ is the frequency in Hz, n is rotor speed in revolution per minutes (rpm) and also known as synchronous speed

$2\pi f = P\, 2\pi\, n/60$

$f = Pn/60$ or $n = 60\, f / P$

In fact, for a given frequency and given number of poles, the speed is fixed. For producing a frequency of 60 Hz, alternator will have to run at speeds given in table below

| No. of poles | 2 | 4 | 6 | 12 | 24 | 36 |
|---|---|---|---|---|---|---|
| Speed (r.p.m.) | 3600 | 1800 | 1200 | 600 | 300 | 200 |

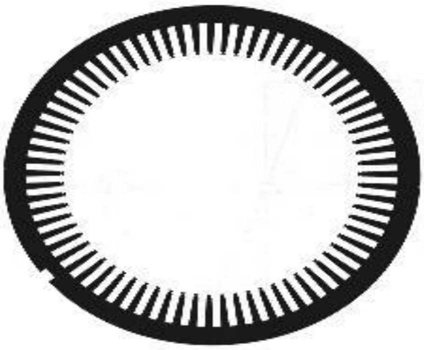
## 4. AC Winding Design

There are two types of windings: Concentrated Windings and Distributed windings.

- Concentrated windings are wound together in series to form one multi-turn coil and all the turns have the same magnetic axis. For example primary and secondary windings of a transformer, field windings for salient-pole synchronous machines.
- Distributed windings are arranged in several full-pitch or fractional-pitch coils. These coils are then housed in slots spread around the air-gap periphery to form phase winding. For examples are stator and rotor windings of induction machines and the armatures of both synchronous and D.C. machines.

## 5. Distributed Three Phase Windings

The stator of a synchronous machine consists of a laminated steel core and a three phase winding. Stator is laminated to minimize loss due to eddy currents. The laminations are stamped out in complete rings. The laminations are insulated from each other and have spaces between them for allowing the cooling air to pass through. The slots for housing the armature conductors lie along the inner periphery of the core and are stamped out at the same time when laminations are formed. Figure 29-a, shows a stator lamination of a synchronous machine that has a number of uniformly distributed slots. Also Different shapes of the armature slots are shown in Fig. 29-b.



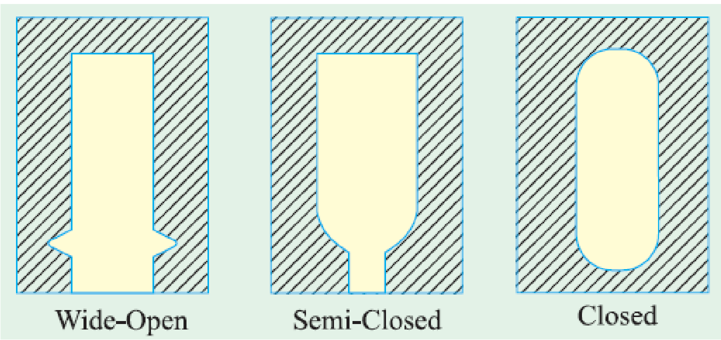



Fig. 29 (a) stator lamination (b) Types of slots

Fig. 30, Stator (armature) construction

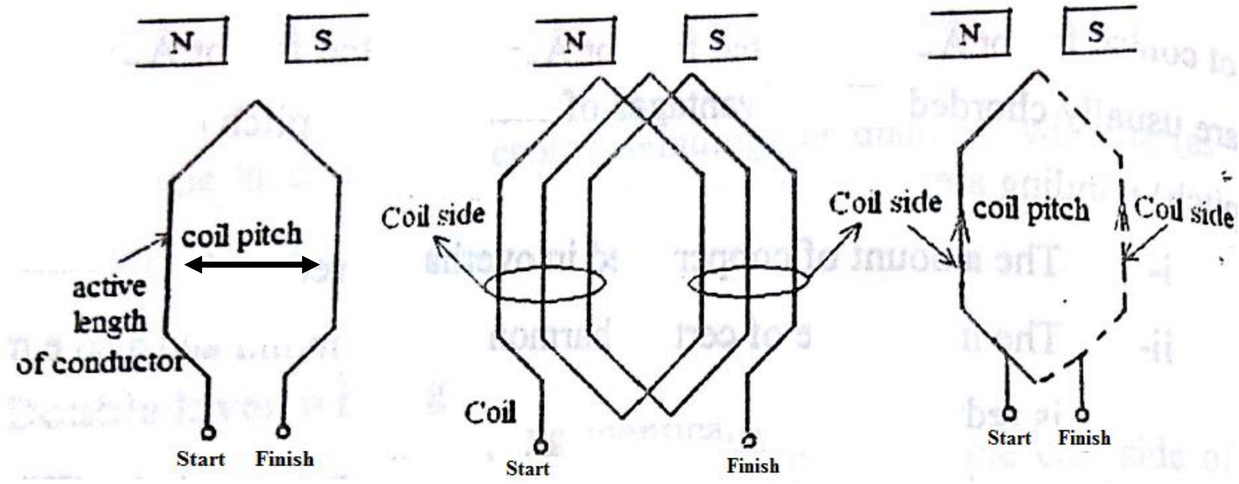
The wide-open type slot has the advantage of permitting easy installation of form-wound coils and their easy removal in case of repair. But it has the disadvantage of distributing the air-gap flux into bunches or tufts, that produce ripples in the wave of the generated e.m.f. The semi-closed type slots are better in this respect, but do not allow the use of form-wound coils. The wholly-closed type slots or tunnels do not disturb the air-gap flux but **(*i*)** they tend to increase the inductance of the windings **(*ii*)** the armature conductors have to be threaded through, thereby increasing initial labor and cost of winding and **(*iii*)** they present a complicated problem of end connections. Hence, they are rarely used.

The 3-phase armature coils are to be laid in these slots and connected in such a way that the current in each phase winding would produce a magnetic field in the air gap around the stator periphery as closely as possible the ideal sinusoidal distribution.

Before dealing with the armature (stator) winding, there are some basic expressions or terms must be defined first;

| | |
|---|---|
| • Conductor | The active length of wire in slot |
| • Turn | Two conductors separated by one pole-pitch or less and connected in series to form one turn |
| • Coil | A coil may consist of single turn or multi turns |
| • Coil Side | Each coil consists of two coil sides that are placed in two different slots at a pole pitch apart or less |
| • Coil Pitch | Is measured in slot pitches between two coil sides |
| • Winding | Is the connection of several coils |

Also these terms are shown in the following figure.




one turn coil    multi-turn coil    multi-turn symbol

Fig. 31, Representation of coil

The following symbol will be used:

| | |
|---|---|
| S | the total number of slots |
| 2P | the total number of poles |
| $\tau$ (tau) | pole pitch = S/2P |
| Q(Coil Group) | number of slots per pole per phase = $\tau$/3 = S/6P |
| $\alpha$ | slot angle in electrical degrees = 180/$\tau$ = 360 P/S |

The armature windings most commonly used for 3-phase alternators can be classified based on **type of windings** into 3 main groups;

***i) Lap Winding***

***ii) Wave Winding***

***iii) Mush Winding***

The armature windings can be classified based on **pitch of coil** into 2 main groups;

***i) Full Pitched Winding*** ***ii) Short Pitched Winding***

The armature windings can be classified based on **number of layers** into 2 main groups;

**(*i*) *single-layer winding*** **(*ii*) *double-layer winding***

Single layer has one coil-side per slot, but double layer type has two coil-sides (one coil) in each slot as shown in Fig. 32.

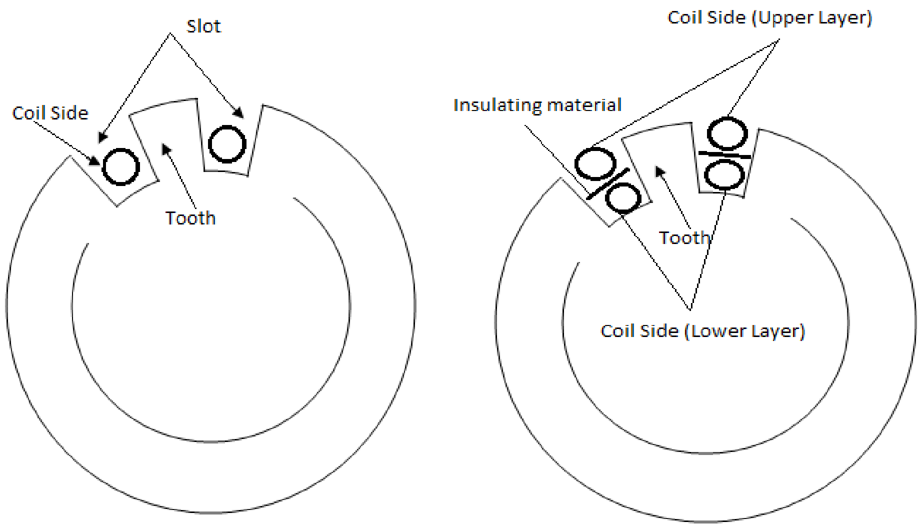



Fig. 32, Difference between single- and double-layer windings

### 5.1 Single-layer winding

In this type of winding, the whole of the slots are occupied by one coil side. This type of winding allows the use of semi-enclosed and closed type of slots. The number of coils used is half of the total number of slots. Single layer winding have been widely used in the past in synchronous and induction machines, but nowadays they are rather seldom used. Single layer winding is used in low power rating machines. A great number of machines with single-layer winding are still in operation.

#### 5.1.1 Three-phase Single-layer Concentric Windings

Concentric windings are single layer winding that use concentric type of coils. The coil span (coil pitch) of the individual coils is different. The coil span for some coils is more than the pole pitch, while the span of others is equal to or less than the pole pitch.

Consider a machine with a stator has 24 slots which gives 4 poles.

The pole pitch is $\tau = S/2P = 24/4 = 6$ and $Q = \tau/3 = 6/3 = 2$ (coil group)

if we consider the three phases are named A, B and C. Then under each pole, there are 2 coils belong to phase A and another two coils belong to phase B and other two coils belong to phase C.

Consider the coils of phase A (which has 2 slots under each pole) are equipped in $S_1$ and $S_2$ and the current direction was upward under the first pole. The coil group of phase A under the 2nd pole will be after pole pitch. i.e 1+6 = 7 and 2+6 = 8. Therefore, the current direction will be reversed (down ward) in the other two slots $S_7$ and $S_8$

The second phase B is similar to phase A, but it is shifted by 120 electric degree. we need to convert 120 electric degrees to slots as follows

the slot angle $\alpha = 180/\tau = 180/6 = 30°$ and slots between phases = 120/30 = 4 slots

So the starting of phase B will be 4 slots after the starting of phase A i.e. $S_5$ and $S_6$

The third phase C will be 4 slots after the starting of phase B i.e. $S_9$ and $S_{10}$.

The current direction in the three-phase winding is governed by the three-phase shape shown in Fig. 33.

At instant T1, phase A is +ve max and phases B & C are -ve with half the max
At instant T2, phase C is -ve max and phases A & B are +ve with half the max
At instant T3, phase B is +ve max and phases A & C are -ve with half the max
At instant T4, phase A is -ve max and phases B & C are +ve with half the max
At instant T5, phase C is +ve max and phases A & B are -ve with half the max
At instant T6, phase B is -ve max and phases A & C are +ve with half the max

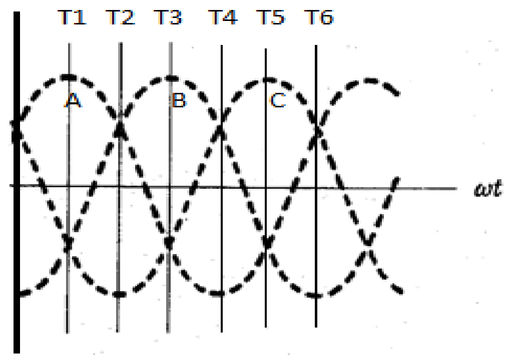



Fig. 33 Three-phase current waveform

These six instants can be summarized in the following table:

| phase / instant | A | B | C |
|---|---|---|---|
| T1 | +ve Max | -ve half Max | -ve half Max |
| T2 | +ve half Max | +ve half Max | -ve Max |
| T3 | -ve half Max | +ve Max | -ve half Max |
| T4 | -ve Max | +ve half Max | +ve half Max |
| T5 | -ve half Max | -ve half Max | +ve Max |
| T6 | +ve half Max | -ve Max | +ve half Max |

Based on T1 instant, the concentric winding is shown in Fig. 34.

In concentric winding, the inner coil has a pitch 7-2 = 5 slots (less than the pole pitch).

But the outer coil has a pitch 8-1 = 7 slots (greater than the pole pitch).

Although the coils of concentric winding differ in pitch, these windings are essentially always full-pitch windings because the phase zones of these windings do not intersect.

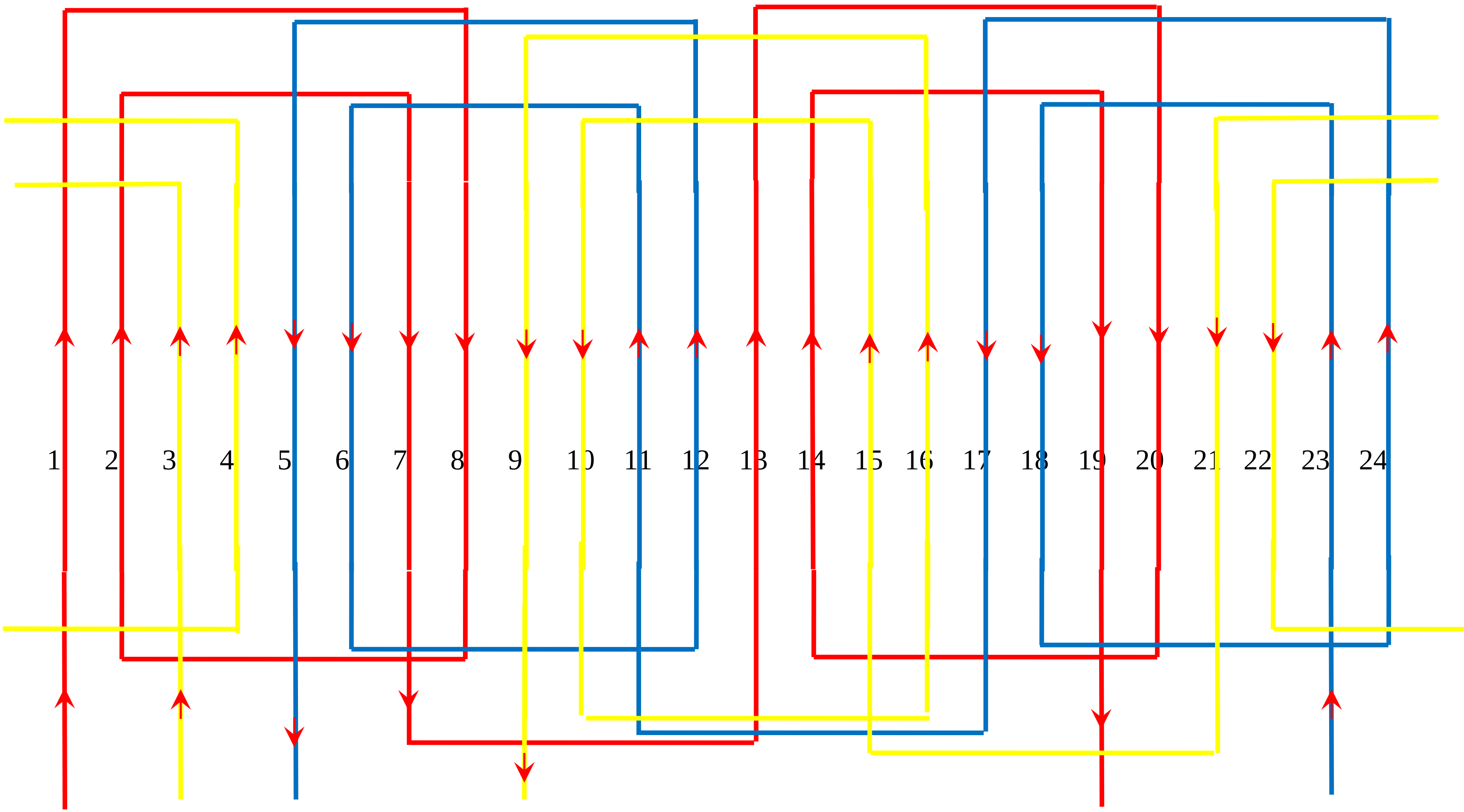



Fig. 34 Single layer concentric windings

### 5.1.2 Three-phase Single-layer Diamond Windings

All the coils of the diamond windings have equal width and similar in shape and may be wound on a common form.

Fig. 35 shows the diamond winding where all coils are full pitched as the coil pitch equal the pole pitch.

The disadvantage of single layer winding is overhang of coils which leads to:

- increase the length of coils, large amount of copper and increase the resistance and inductance of winding. This means increase the machine losses.
- reduce the cooling and therefore cause a poor performance

To improve the performance characteristics of the machine, double layer winding is used.

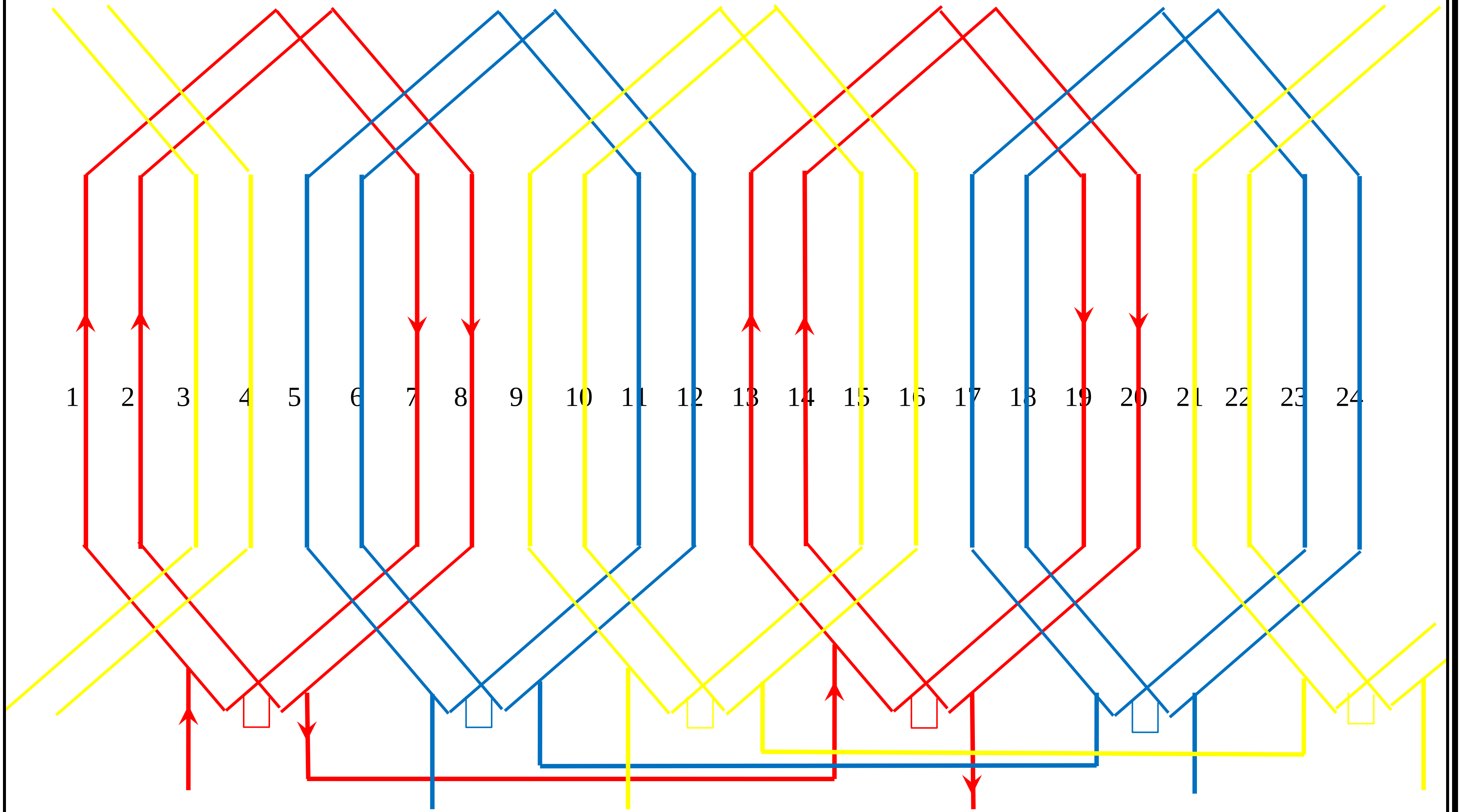



Fig. 35 Three-phase Single-layer Diamond windings

### 5.2 Double-layer winding

Two important points regarding this winding should be noted:

**(*a*)** Ordinarily, the number of slots in stator (armature) is a multiple of the number of poles and the number of phases. Thus, the stator of a 4-pole, 3-phase alternator may have 12, 24, 36, 48 etc. slots all of which are seen to be multiple of 12 (*i.e*. 4 × 3).

**(*b*)** The number of stator slots is equal to the number of coils (which are all of the same shape). In other words, each slot contains two coil sides, one at the bottom of the slot and the other at the top. The upper coil side is plotted as solid line but the bottom coil side is represented by dashed line. The coils overlap each other, just like shingles on a roof top.

#### 5.2.1 Full-pitch winding (integral slot)

The fundamental principle of such a winding is illustrated in Fig. 36, which shows a double-layer, full-pitch winding for a four-pole generator (2P). There are 24 slots (S) in all, giving 6 slots per pole **(pole pitch τ)** or 2 slot/pole/phase **(coil group Q)**.

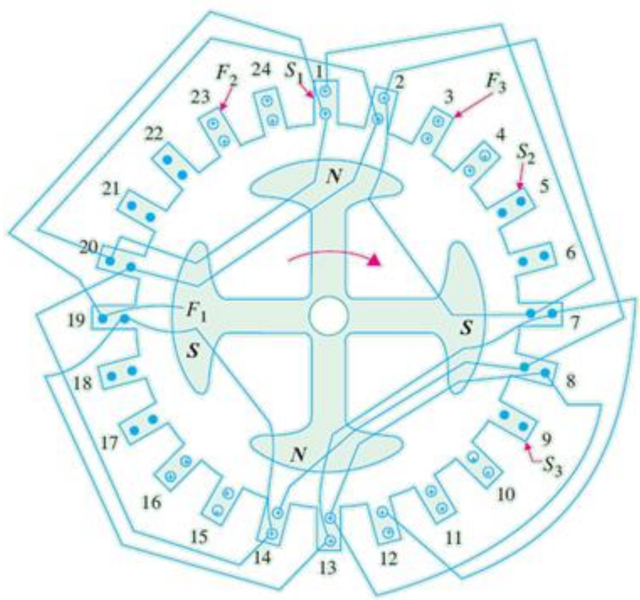



Fig. 36, Three-phase, double layer, full-pitch lab windings

$$Q = \frac{S}{2P \times 3} = \frac{S}{6P} = \frac{24}{6 \times 2} = 2$$

$$\tau = \frac{S}{2P} = \frac{24}{4} = 6$$

To get maximum e.m.f., the two sides of a coil should be one pole-pitch apart *i.e.* coil pitch (**$\tau_c$**) should be equal to one pole pitch. In other words, if one side of the coil is under the centre of a *N*-pole, then the, other side of the same *coil* should be under the centre of *S*-pole *i.e.* 180° (electrical) apart. In that case, the e.m.f's induced in the two sides of the coil are added together. It is seen from Fig. 28, that *R* phase starts at slot No. 1 (SR = start red), and finishes at slot No. 7. The two slots 1 and 7 being one pole-pitch or 180° (electrical) apart. Since Q=2, this means there is another coil belong to red phase and located in slot 2 and end at slot 8. These two red coils are connected in series.

Assuming RYB sequence, the *Y*-phase starts 120° afterwards. The slot angle (α) is obtained as:

$$\alpha = \frac{180}{\tau} = \frac{180}{6} = 30°$$

This means the angle 120° equivalent to 4 slots. The Y phase starts at slot 5 *i.e.* four slots away from the start of *R*-phase and finishes at 11. The other yellow coil is located in slot 6 and finish at 12. Similarly, *B*-phase starts from slot No. 9 *i.e.* four slots away from the start of *Y*-phase, and finishes at slot No. 15. The other coil of B phase is located at slot 10 and finish at slot 16.

### 5.2.2 Short-pitch winding (integral slot)

So far we have discussed full-pitched coils *i.e.* coils having span which is equal to one pole-pitch *i.e.* spanning over 180° (electrical).

As shown in Fig. 37, if the coil sides are placed in slots 1 and 7, then it is full-pitched. If the coil sides are placed in slots 1 and 6, then it is short-pitched or fractional-pitched because coil span is equal to 5/6 of a pole-pitch. It falls short by 1/6 pole-pitch or by 180°/6 = 30°.

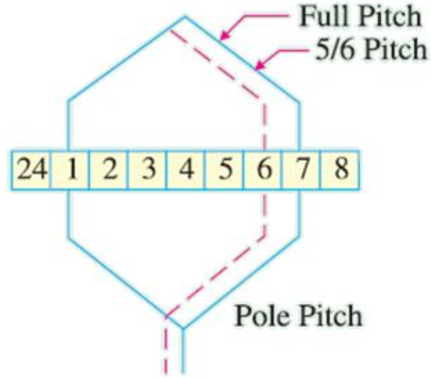



Fig. 37, Chorded armature winding

Consider a double-layer, short-pitch winding for a four-pole generator (2P=4) and 24 slots (S=24). If the coil pitch (**$\tau_c$ = 5)**, the developed diagram of such kind of winding is shown in Fig. 38.

Short-pitched coils are deliberately used because of the following advantages:

**1.** They save copper of end connections.

**2.** They improve the wave-form of the generated e.m.f. *i.e.* the generated e.m.f. can be made to approximate to a sine wave more easily and the distorting harmonics can be reduced or totally eliminated.

**3.** Due to elimination of high frequency harmonics, eddy current and hysteresis losses are reduced thereby increasing the efficiency.

But the disadvantage of using short-pitched coils is that the total voltage around the coils is somewhat reduced. Because the voltages induced in the two sides of the short-pitched coil are slightly out of phase, their resultant vectorial sum is less than their arithmetical sum.

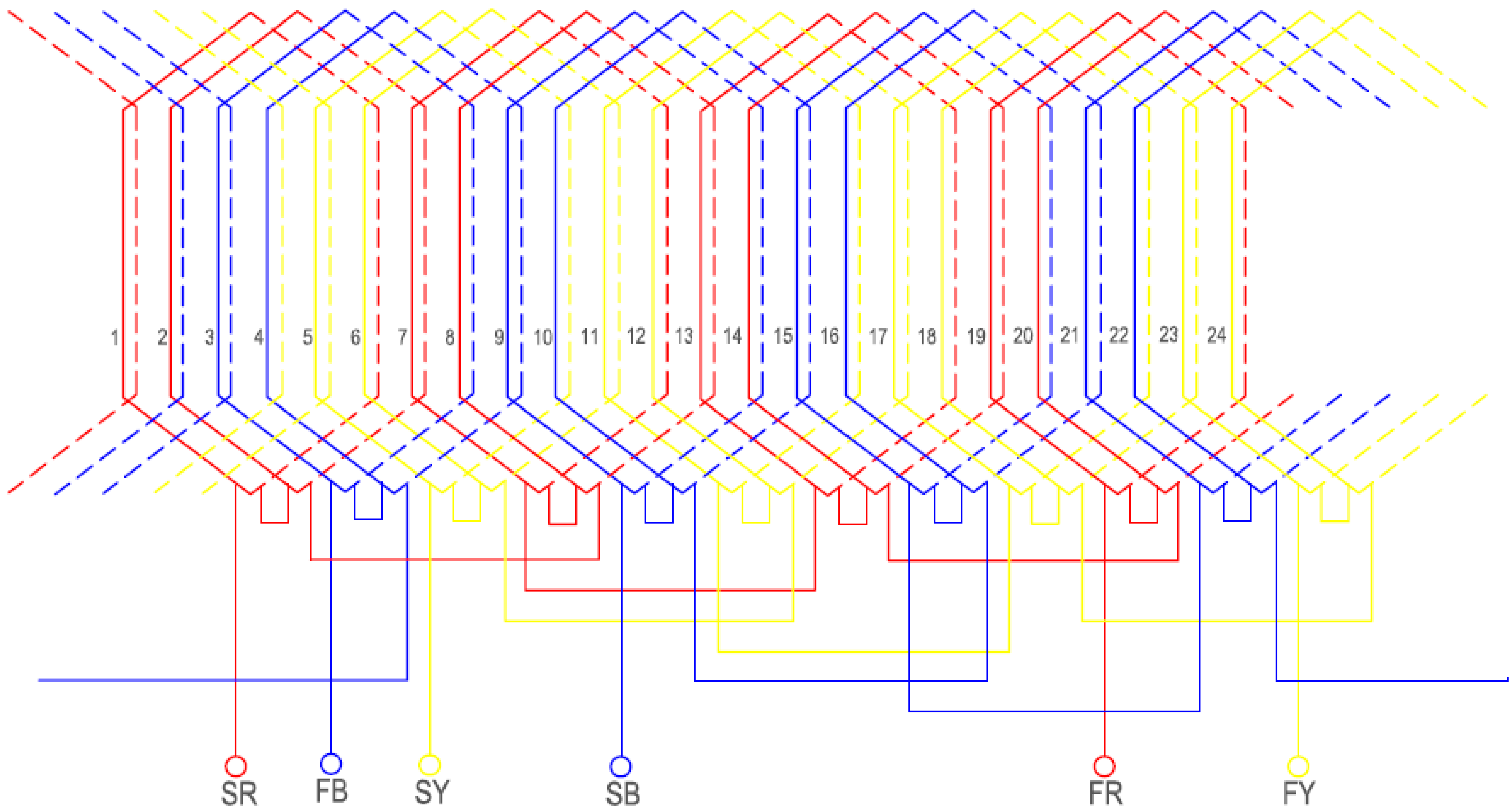



Fig. 38, Three-phase, double layer, short-pitch lab windings

### 5.3 Chording factor ($K_c$)

The pitch factor ($K_p$) or chording factor ($K_c$), which is always less than unity, is defined as:

$$K_c = \frac{vectorial\ sum\ of\ the\ induced\ emf's\ per\ coil}{arithmatic\ sum\ of\ the\ induced\ emf's\ per\ coil}$$

Let $E_S$ be the induced e.m.f. in each coil side. If the coil were full-pitched *i.e.* if its two sides were one pole-pitch apart, then total induced e.m.f. in the coil would have been = $2E_S$ as shown in Fig. 39-a. If it is short-pitched by 30° (elect.) then as shown in Fig. 39-b, their resultant is $E$ which is the vectorial sum of two voltage 30° (electrical) apart ($E = 2E_s \cos(15) = 0.966$ which is less than unity.

$$K_c = \frac{vector\ sum}{arithmatic\ sum} = \frac{2\ E_s \cos(15)}{2\ E_s} = \cos(15)$$

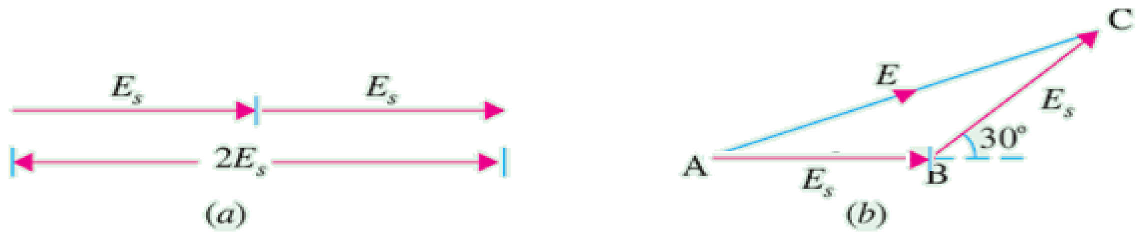



Fig. 39, Chording factor

Generally speaking, if the coil span is chorded by $k$ slots.

Therefore the chording angle ($\gamma$) is given by,

$$\gamma = k\ \alpha$$

Therefore, chording factor ($K_c$) is defined by

$$K_c = \cos\left(\frac{\gamma}{2}\right)$$

**Example (13)**

Calculate the chording factor for the under-given windings: **(a)** 36 stator slots, 4-poles, coil-span, 1 to 8 **(b)** 72 stator slots, 6 poles, coils span 1 to 10

**Case (a)** as shown in Fig. 40-a, pole pitch, $\tau = 36/4 = 9$, coil span $\tau_c = 8-1=7$

There is chording by $\tau - \tau_c = 9-7 = 2$ slots

The slot angle $\alpha = 180/9 = 20$ electrical degrees

The chording angle γ = 2*20 = 40 electrical degrees

$$K_c = \cos\left(\frac{\gamma}{2}\right) = \cos\left(\frac{40}{2}\right) = 0.94$$

**Case (b)** as shown in Fig. 40-b, pole pitch, $\tau$ = 72/6 = 12, coil span $\tau_c$ = 10-1=9

There is chording by $\tau - \tau_c$ = 12-9 = 3 slots

The slot angle α = 180/12 = 15 electrical degrees

The chording angle γ = 3*15 = 45 electrical degrees

$$K_c = \cos\left(\frac{\gamma}{2}\right) = \cos\left(\frac{45}{2}\right) = 0.924$$

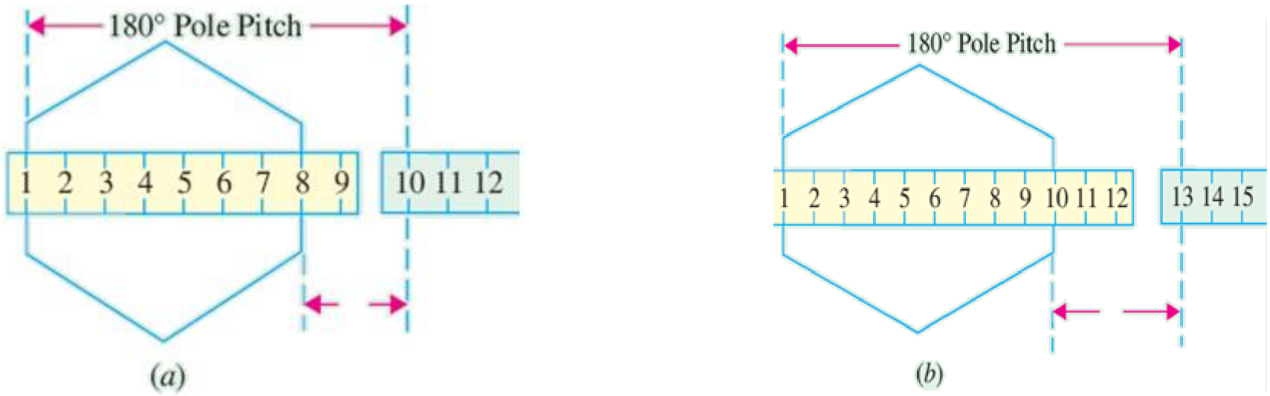



Fig. 40, developed winding diagram for example 1

### 5.4 Distribution Factor $K_d$

As seen in the developed winding, that in each phase, coils are not concentrated or bunched in one slot, but are distributed in a number of slots to form polar groups under each pole. These coils/phase are displaced from each other by a certain angle (slot angle α). The result is that the e.m.f's induced in coil sides constituting a polar group are not in phase with each other but differ by an ( α ) angle.

Consider a 3-phase double-layer winding for a 4-pole alternator. It has a total of 36 slots *i.e.* $\tau$ = 9 slots/pole. Obviously, there are Q = 3 slots / pole / phase. For example, coils 1, 2 and 3 belong to *R* phase. Now, these three coils which constitute one polar group are not bunched in one slot but in three different slots. Angular displacement between any two adjacent slots = 180°/9 = 20° (elect.). If the three coils were bunched in one slot, then total e.m.f. induced in the three sides of the coil would be the arithmetic sum of the three e.m.f.s. *i.e.* = 3 $E_S$ as shown in Fig. 41-a, where $E_S$ is the e.m.f. induced in one coil side.

Since the coils are distributed, the individual e.m.fs. have a phase difference of 20° with each other. Their vector sum is as shown in Fig. 41-b.

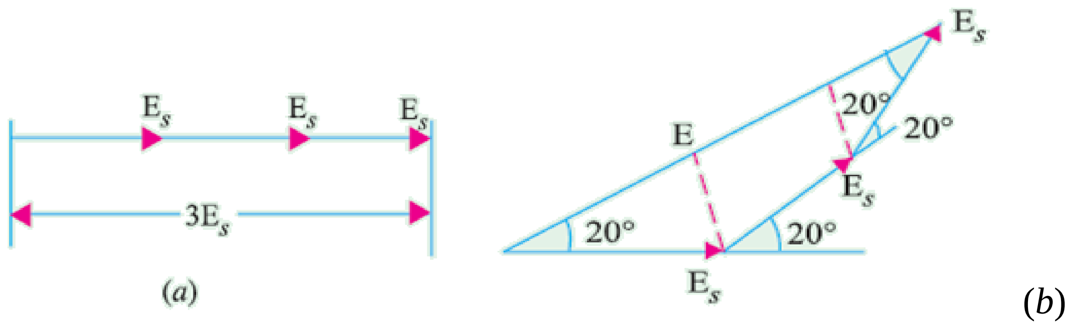



Fig. 41, Distribution factor for coil group of 3 coils in series

The distribution factor ($K_d$) is defined as

$$K_d = \frac{emf\ with\ distributed\ winding}{emf\ with\ concentrated\ winding}$$

Consider a Q coil group shown in Fig. 42, the polygon ABCDE is considered as part of a circle of radius *r*

$AB = E_S = 2r \sin \alpha/2$

Arithmetic sum is = $QE_S = Q \times 2r \sin \alpha/2$

Their vector sum= $AE = E_r = 2r \sin Q\alpha/2$

$$K_d = \frac{vector\ sum}{arithmatic\ sum} = \frac{2r\ sin\left(\frac{Q\alpha}{2}\right)}{Q\ 2r \sin\left(\frac{\alpha}{2}\right)} = \frac{sin\left(\frac{Q\alpha}{2}\right)}{Q\ \sin\left(\frac{\alpha}{2}\right)}$$

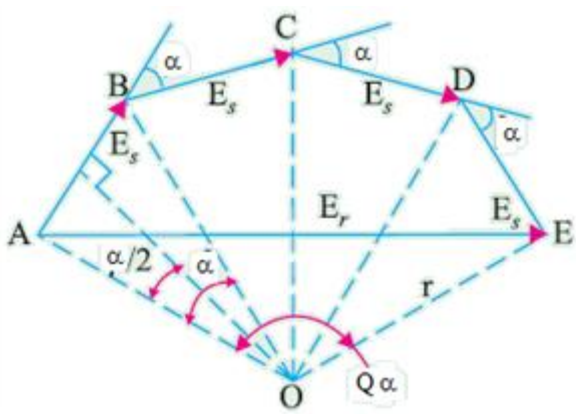



Fig. 42, distribution factor $K_d$

The value of distribution factor ($K_d$) of a 3-phase alternator for different number of slots/pole/phase is given in table 2.

Table 2.

| Slots per pole | Q | α° | Distribution factor $k_d$ |
|---|---|---|---|
| 3 | 1 | 60 | 1.000 |
| 6 | 2 | 30 | 0.966 |
| 9 | 3 | 20 | 0.960 |
| 12 | 4 | 15 | 0.958 |
| 15 | 5 | 12 | 0.957 |
| 18 | 6 | 10 | 0.956 |
| 24 | 8 | 7.5 | 0.955 |

**<u>Example (14):</u>**

Calculate the distribution factor for a 36-slots, 4-pole, double-layer three-phase winding.

$$\tau = \frac{S}{2P} = \frac{36}{4} = 9$$

$$Q = \frac{S}{6\,P} = \frac{36}{12} = 3$$

$$\alpha = \frac{180}{\tau} = \frac{180}{9} = 20$$

$$K_d = \frac{sin\left(\frac{Q\alpha}{2}\right)}{Q\ \sin\left(\frac{\alpha}{2}\right)} = \frac{\sin\left(\frac{3\times 20}{2}\right)}{3\sin\left(\frac{20}{2}\right)} = \frac{\sin(30)}{3\sin(10)} = 0.96$$

**<u>Example (15):</u>**

A part of an alternator winding consists of six coils in series, each coil having an e.m.f. of 10 V r.m.s. induced in it. The coils are placed in successive slots and between each slot and the next, there is an electrical phase displacement of 30°. Find graphically or by calculation, the distribution factor of the six coils in series.

Q = 6 and α = 30

$$K_d = \frac{sin\left(\frac{Q\alpha}{2}\right)}{Q\ \sin\left(\frac{\alpha}{2}\right)} = \frac{\sin\left(\frac{6\times 30}{2}\right)}{6\sin\left(\frac{30}{2}\right)} = \frac{\sin(90)}{6\sin(15)} = 0.2588$$

### 4.5 Fractional slot winding

In some cases the number of slots per pole (τ) as well as the number of slots per pole per phase (Q) are not integer

Consider a 3-phase alternator with 27 slots and 6 poles. If the coil span is 4 slots, the developed winding of this alternator is shown in Fig. 43.

$$Q = \frac{S}{6\,P} = \frac{27}{6\times 3} = \frac{3}{2}$$

As seen before, when the 3-phase sequence was RYB, the armature windings are placed physically as RBY. Construct Table 3 such that has 3 main columns (the First for R-phase, the second for B-phase and the third for Y-phase). Each column is divided to sub columns equal the numerator of Q (in our case is 3). The number of rows must equal the number of poles. Starting with slot 1 in the first cell, then skip 2 cells (denominator of Q) put slot 2 and so on until the 27 slots are distributed over the table cells.

Table 3.

| | R | | | B | | | Y | | |
|---|---|---|---|---|---|---|---|---|---|
| **N** | 1 | | 2 | | 3 | | 4 | | 5 |
| **S** | | 6 | | 7 | | 8 | | 9 | |
| **N** | 10 | | 11 | | 12 | | 13 | | 14 |
| **S** | | 15 | | 16 | | 17 | | 18 | |
| **N** | 19 | | 20 | | 21 | | 22 | | 23 |
| **S** | | 24 | | 25 | | 26 | | 27 | |

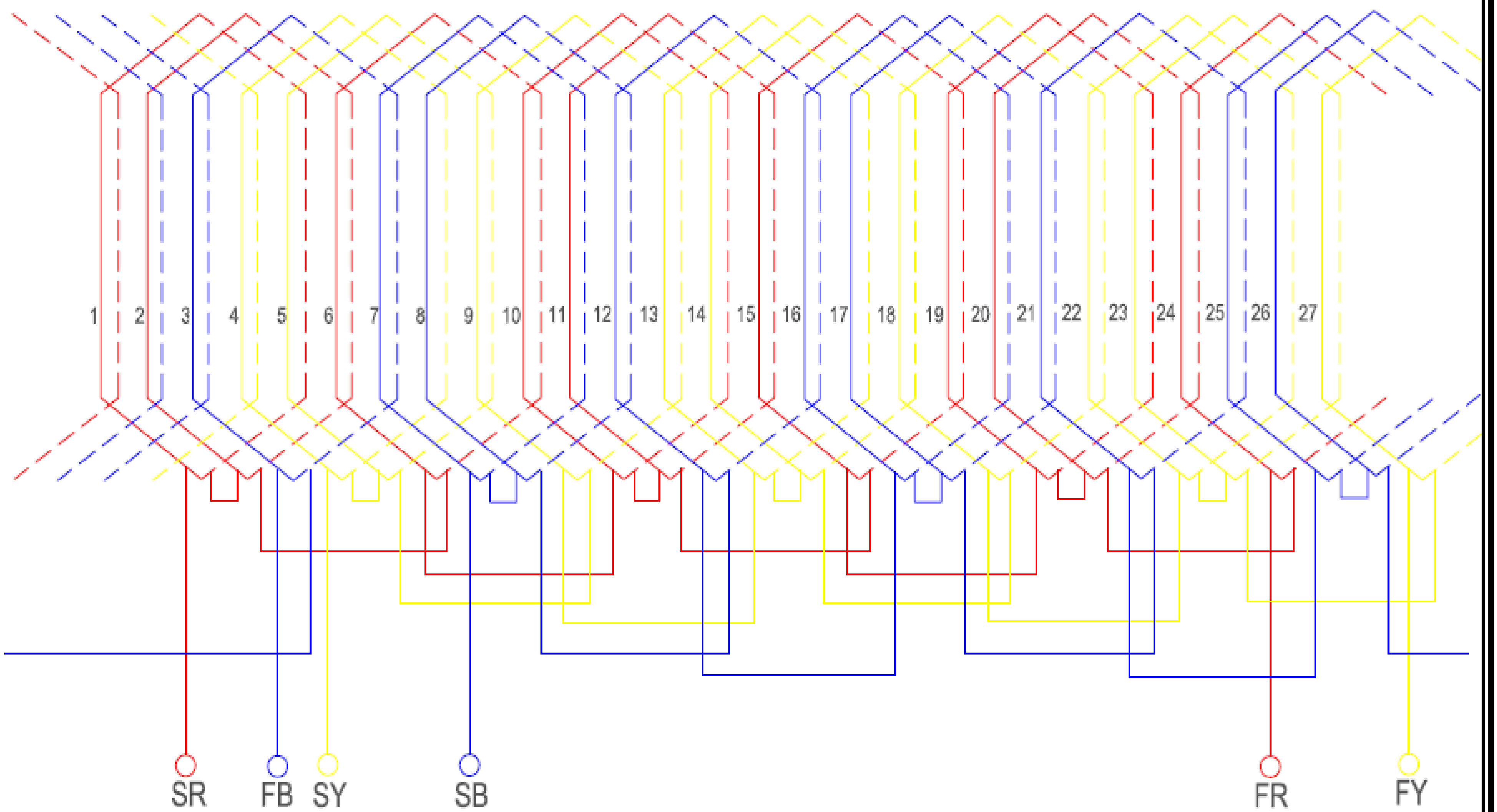



Fig. 43, fractional slot double-layer winding

**Example (16):**

Consider a 3-phase alternator with 21 slots and 4 poles. If the coil span is 5 slots, draw the developed winding of this alternator.

$$Q = \frac{S}{6\,P} = \frac{21}{6 \times 2} = \frac{7}{4}$$

Construct the table for the fractional slot winding as Table 4.

Table 4

| | R | | | | | | | B | | | | | | | Y | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **N** | 1 | | | | 2 | | | | 3 | | | | 4 | | | | 5 | | | | 6 |
| **S** | | | | 7 | | | | 8 | | | | 9 | | | | 10 | | | | 11 | |
| **N** | | | 12 | | | | 13 | | | | 14 | | | | 15 | | | | 16 | | |
| **S** | | 17 | | | | 18 | | | | 19 | | | | 20 | | | | 21 | | | |

The developed winding diagram is given in Fig. 44.

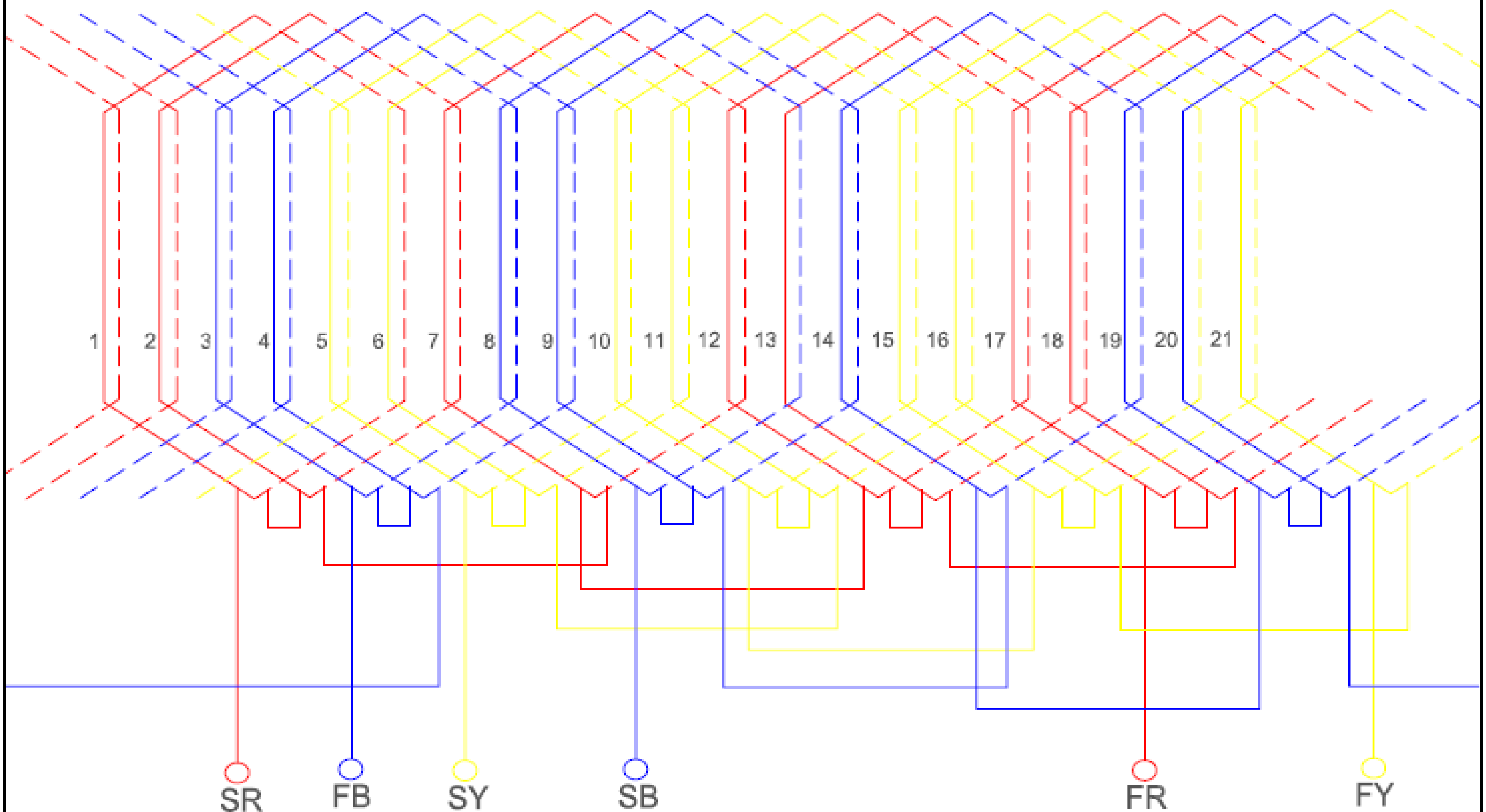



Fig. 44, fractional slot, double layer winding

**Example (17):**

Consider a 3-phase alternator with 30 slots and 4 poles. If the coil span is 7 slots, draw the developed winding of this alternator.

$$Q = \frac{S}{6\,P} = \frac{30}{6 \times 2} = \frac{5}{2}$$

Construct the table for the fractional slot winding as Table 5.

Table 5

| | R | | | | | B | | | | | Y | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **N** | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | 8 |
| **S** | | 9 | | 10 | | 11 | | 12 | | 13 | | 14 | | 15 | |
| **N** | 16 | | 17 | | 18 | | 19 | | 20 | | 21 | | 22 | | 23 |
| **S** | | 24 | | 25 | | 26 | | 27 | | 28 | | 29 | | 30 | |

The developed winding diagram is given in Fig. 45.

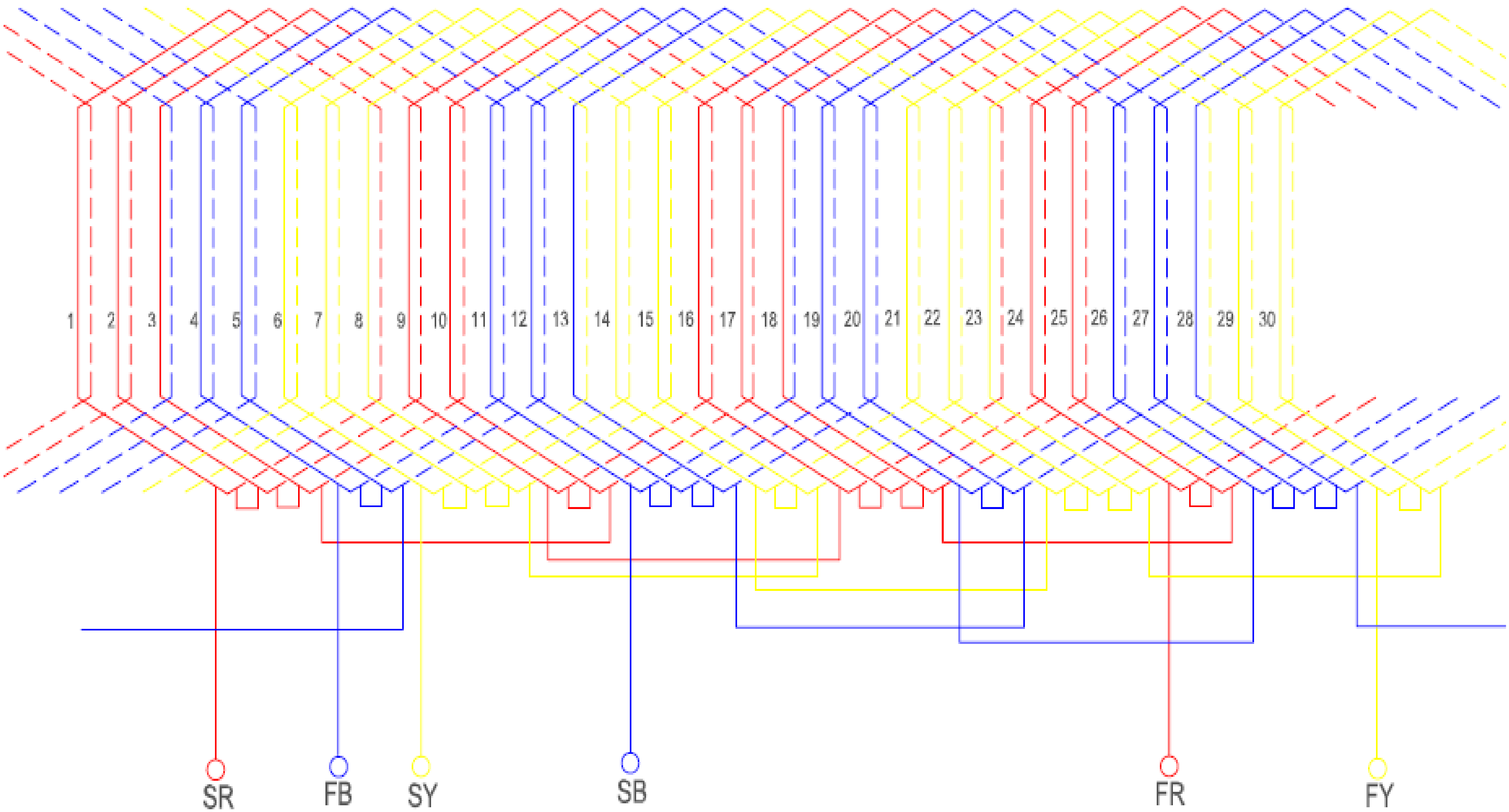



Fig. 45, fractional slot, double layer winding

## 5. Equation of Induced E.M.F

Consider the elementary alternator shown in the following figure:

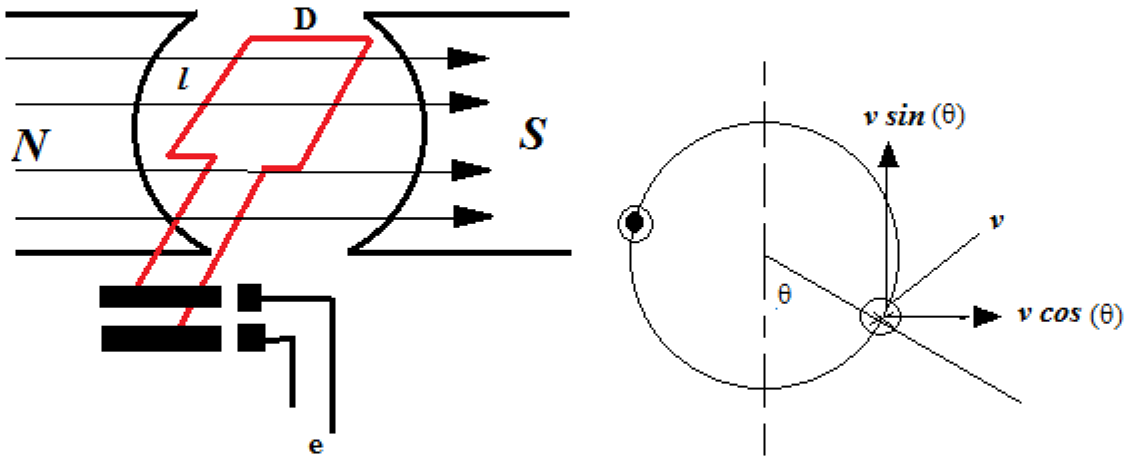



Fig. 46, Induced emf

D is the diameter of the circular path,

*l* is the effective conductor length (meter)

The induced emf in a conductor moving in a magnetic field is given by:

$$e = B\ l\ v$$

where, B is the flux density (Wb/m$^2$)

*v* is the velocity (m/s)

Since the turn is consists of 2 conductors, therefore the induced emf in on turn is

$$e = 2\ B\ l\ v$$

At the instant when the coil at a position defined by angle (θ), the tangential velocity for circular path has two components:

- Horizontal component (*v* cos(θ)) that does not cause emf as it is parallel to the field.
- Vertical component (*v* sin(θ)) and it is the effective component to produce an emf.

$$e = 2\ B\ l\ v \sin(\theta)$$

the velocity and flux density are given as:

$$v = \pi\ D\ \frac{n}{60} \qquad \&\ B = \frac{\varphi}{l \times D}$$

where n is the rotational speed in rpm

substituting the values of v and B in the emf equation,

$$e = 2\frac{\varphi}{l \times D} l\,\pi\,D \frac{n}{60}\sin(\theta) = 2\pi\,\varphi \frac{n}{60}\sin(\theta)$$

As we know, the frequency of the induced emf is given as:

$$f = \frac{n\,P}{60} \rightarrow \frac{n}{60} = \frac{f}{P}$$

$$e = 2\pi\,\varphi \frac{f}{P}\sin(\theta)$$

For total turns per phase $N_{ph}$, the emf becomes

$$e = 2\pi\,N_{ph}\varphi \frac{f}{P}\sin(\theta)$$

the maximum emf is obtained at θ = 90, then

$$e_{max} = 2\pi\,N_{ph}\varphi \frac{f}{P}$$

the rms value of the emf = $e_{max}/\sqrt{2}$, then

$$e_{rms} = \sqrt{2}\pi\,N_{ph}\varphi \frac{f}{P} = 4.44\,N_{ph}\varphi \frac{f}{P}$$

For a machine has number of pair poles (P), will be simply P times the emf given above

$$e_{rms} = 4.44\,P\,N_{ph}\varphi \frac{f}{P} = 4.44\,f\,\varphi\,N_{ph}$$

Considering the effect of the winding factor, then the rms value of the induced EMF per phase (E) is given by:

$$E = 4.44\,f\,\varphi\,N_{ph}K_w$$

Where *f* is the supply frequency in Hz,

Φ is the magnetic flux per pole in webers

$N_{ph}$ is the number of turns per phase

$K_w$ is the winding factor which is defined as $K_c\,K_d$

**Example (18):**

A 3-phase, 16-pole alternator has a star-connected winding with 144 slots and 10 conductors per slot. The flux per pole is 0.03 Wb, Sinusoidally distributed and the speed is 375 r.p.m. Find the frequency and the phase and line e.m.f. Assume the coil span is chorded by 2 slots.

Total number of conductors = 10 * 144 = 1440 conductors

Total number of turns = 1440 / 2 = 720 turns

Number of turns per phase ($N_{ph}$) = 720 / 3 = 240 turns/phase

Since $f = \frac{n\,P}{60} = \frac{375 \times 8}{60} = 50\,Hz$, $\tau = \frac{S}{2\,P} = \frac{144}{2\times 8} =$ 9 slot/pole

$Q = \frac{S}{6\,P} = \frac{144}{6\times 8} =$ 3 slot/pole/phase and $\alpha = \frac{180}{\tau} = \frac{180}{9} =$ 20 electrical degrees

γ = number of chorded slots * α = 2 * 20 = 40 electrical degrees

$$K_c = \cos\left(\frac{\gamma}{2}\right) = \cos\left(\frac{40}{2}\right) = 0.94$$

$$K_d = \frac{sin\left(\frac{Q\alpha}{2}\right)}{Q\ \sin\left(\frac{\alpha}{2}\right)} = \frac{\sin\left(\frac{3\times 20}{2}\right)}{3\sin\left(\frac{20}{2}\right)} = \frac{\sin(30)}{3\sin(10)} = 0.96$$

Winding factor $K_w = K_c\,K_d$ = 0.94 * 0.96 = 0.9024

Phase EMF = 4.44 $f\,\Phi\,N_{ph}\,K_w$ =4.44 * 50 * 0.03 * 240 * 0.9024 = 1442.396 volt

Since the armature winding of the alternator is star connected,

Line EMF = √3 * Phase EMF = √3 * 1442.396 = 2498.3 volt

## 6. Rotating Magnetic Fields

### 6.1 Magnetic Field of a Distributed Phase Winding

The magnetic field distribution of a distributed phase winding can be obtained by adding the fields generated by all the coils of the winding. The diagram given in Fig. 47 shows the profiles of *mmf* and field strength of a single coil in a uniform air gap. If the permeability of the iron is assumed to be infinite, by Ampere's law, the *mmf* across each air gap would be *Ni*/2, where

*N* is the number of turns of the coil and *i* the current in the coil.

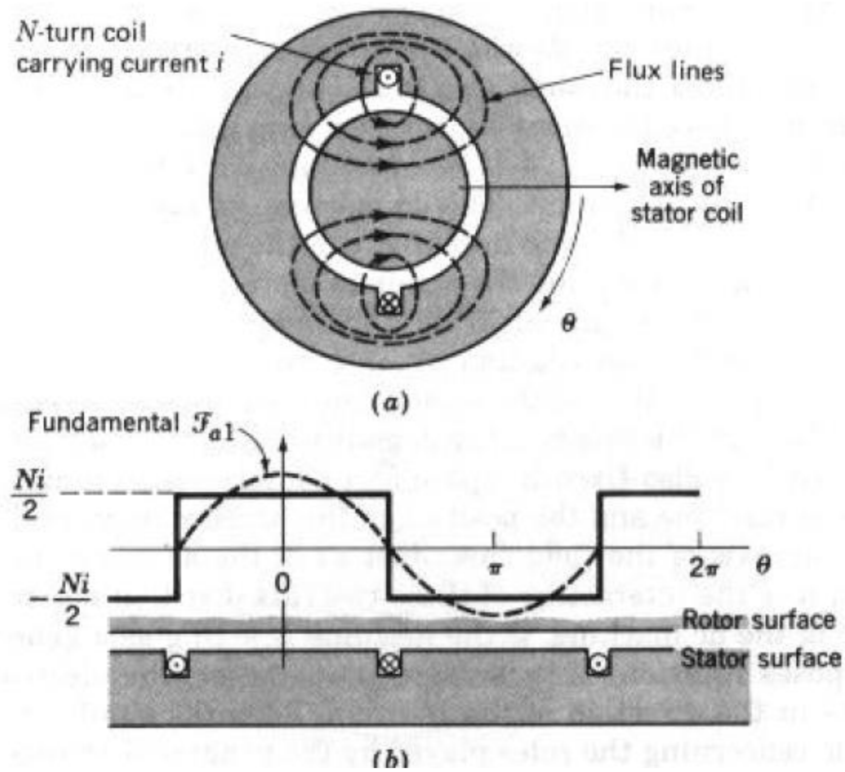



Fig. 47, MMF of a concentrated, full-pitch coil

The *mmf* distribution along the air gap is a square wave. Because of the uniform air gap, the spatial distribution of magnetic field strength is the same as that of *mmf*.

Using Fourier series, the square wave represents the mmf waveform can be expanded to a fundamental component ($F_{a1}$) and odd harmonics. Fundamental component is given as

$$F_{a1} = \frac{4}{\pi}\frac{Ni}{2}\cos(\theta) = \frac{2}{\pi}N\,i\cos(\theta)$$

where $\theta$ is the angular displacement from the magnetic axis of the coil.

When the field distributions of a number of distributed coils are combined, the resultant field distribution is close to a sine wave, as shown in Fig. 40. The fundamental component of the resultant *mmf* can be obtained by adding the fundamental components of these individual coils, and it can expressed as:

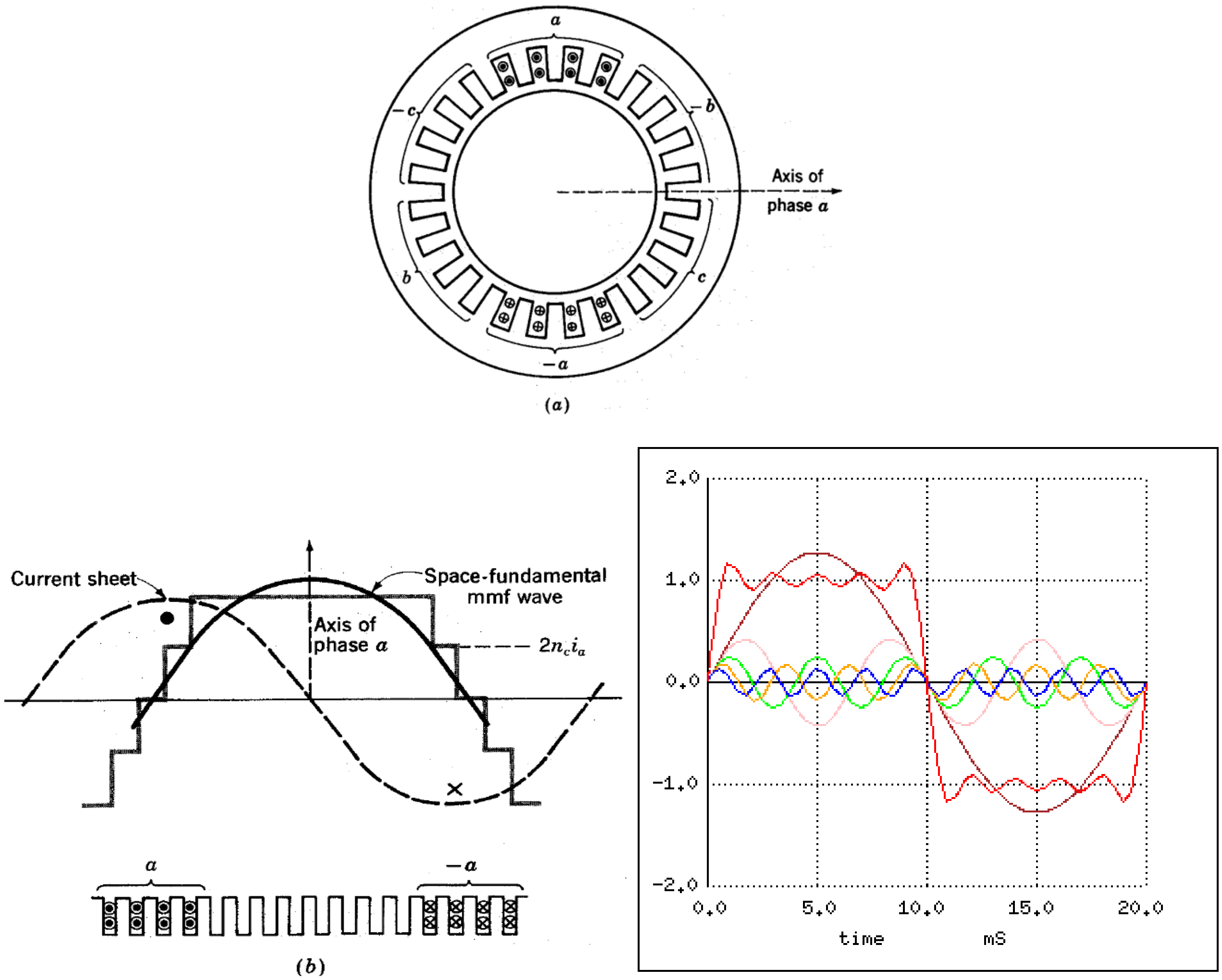



Fig. 48, MMF of one phase armature winding

$$F_{a1} = \frac{2}{\pi}\frac{N_{ph} i_a}{P}\cos(\theta)\ K_w$$

Where, $N_{ph}$ is the total number of turns per phase, which is formed by these coils.

In most cases, the value of ($K_w$ $N_{ph}$) is known as the effective number of turns of the phase winding.

Assume the phase current flowing through phase (*a*) is $i_a = I_m \cos(\omega t)$ , Therefore,

$$F_{a1} = \frac{2}{\pi}\frac{K_w N_{ph} I_m}{P}\cos(\omega t)\cos(\theta)$$

$$F_{a1} = F_m \cos(\omega t)\cos(\theta)$$

Where,

$$F_m = \frac{2}{\pi}\frac{K_w N_{ph} I_m}{P} = \frac{2}{\pi}\frac{K_w N_{ph}}{P}\left(\sqrt{2}\,I_{rms}\right) = 0.9\,\frac{K_w N_{ph}\,I_{rms}}{P}$$

It is clear from the above equation; the *mmf* of a distributed phase winding is a function of both space and time.

A Matlab code is used at 10 different values of time to plot the relation between the position angle (θ) and the generated MMF as follows:

```
Time=0:0.1*pi:2*pi;
Theta=0:0.01*pi:2*pi;
for J=1:20;
    for I=1:201
        Fm(J,I)=50*cos(Time(J))*cos(Theta(I));
    end
end
plot(Theta/pi,Fm(1,:),Theta/pi,Fm(2,:),Theta/pi,Fm(3,:),Theta/pi,Fm(4,:),...
    Theta/pi,Fm(5,:),Theta/pi,Fm(6,:),Theta/pi,Fm(7,:),Theta/pi,Fm(8,:),...
    Theta/pi,Fm(9,:),Theta/pi,Fm(10,:))
grid
```

When plotted at different time instants as shown in Fig. 41, we see that it is a pulsating sine wave. This type of the phase *mmf* as a ***pulsating mmf***.

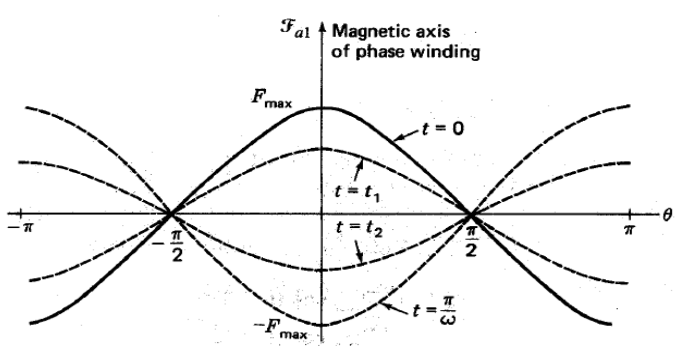


Fig. 49, Pulsating MMF of the phase winding (Continue in next page)

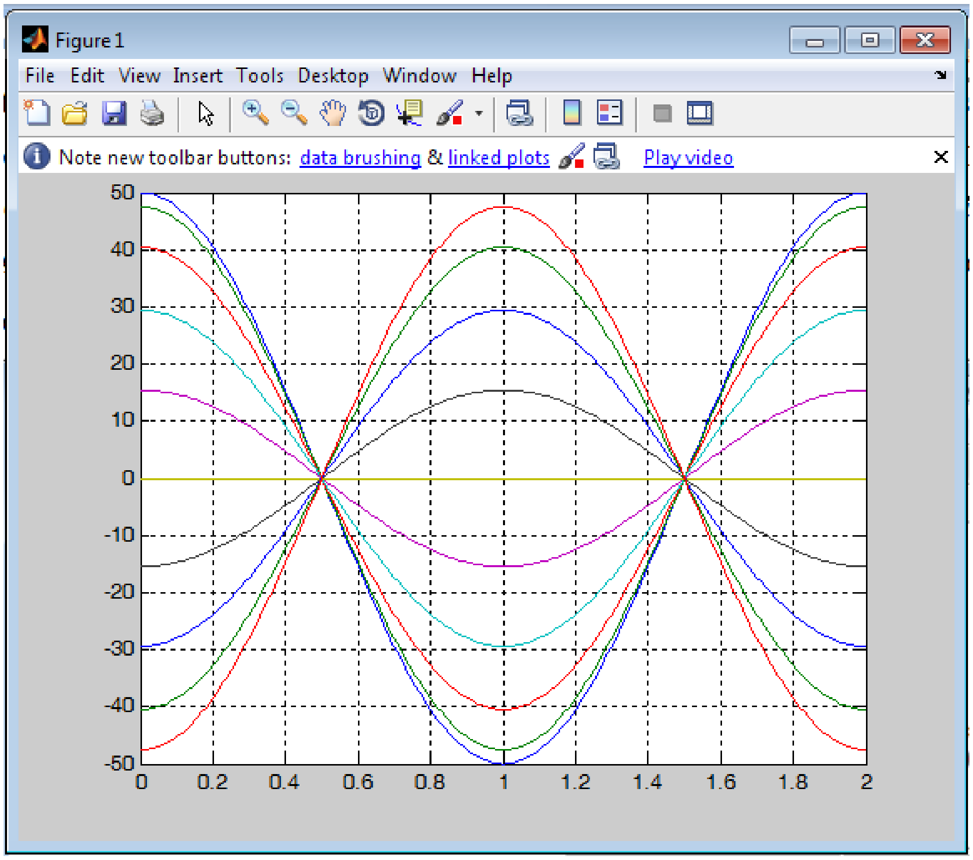



Fig. 19, Pulsating MMF of the phase winding

The $n^{th}$ harmonic components of MMF are given by:

$$F_{an} = F_{mn}\cos(\omega t)\cos(n\theta)$$

where,

$$F_{mn} = \frac{2}{\pi}\frac{K_{wn}N_{ph}}{nP}\left(\sqrt{2}\, I_{rms}\right) = 0.9\,\frac{K_{wn}N_{ph}\, I_{rms}}{nP}$$

As we know,

$$\cos(\alpha)\cos(\beta) = \frac{\cos(\alpha-\beta)+\cos\,(\alpha+\beta)}{2}$$

Therefore,

$$F_{a1} = \frac{F_m}{2}\cos(\theta-\omega t) + \frac{F_m}{2}\cos(\theta+\omega t)$$

$$F_{a1} = F^{+} + F^{-}$$

It can be seen that the first term in the above equation stands for a rotating *mmf* in the $+\theta$ direction and the second term is a rotating *mmf* in the $-\theta$ direction. That is a pulsating *mmf* can be resolved into two rotating *mmf*'s components with the same magnitudes and opposite rotating directions, as shown in Fig. 50. For a machine with uniform air gap, the above analysis is also applicable to the magnetic field strength and flux density in the air gap.

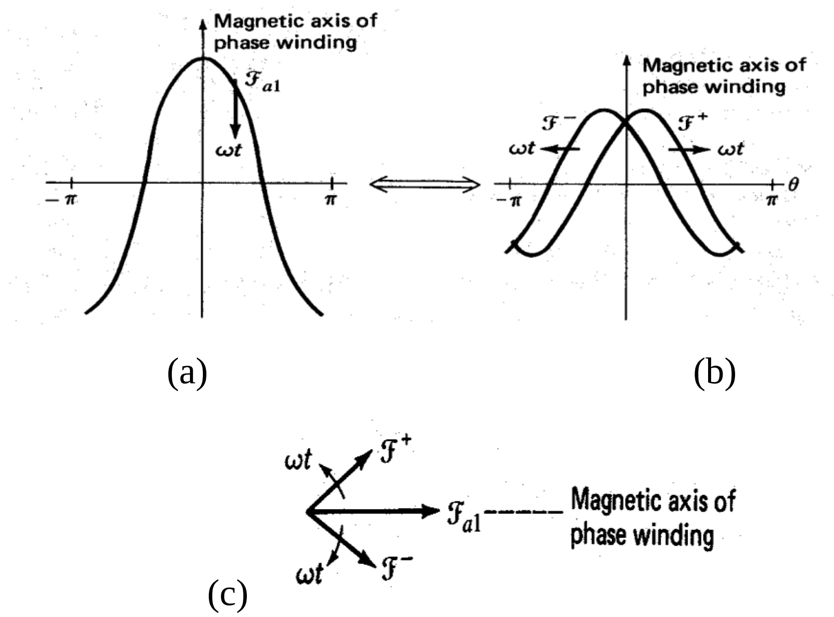



Fig. 50, a) Total mmf per phase, b) total mmf decomposed into two traveling components, c) phasor decomposition

## 6.2 Magnetic Field of Three Phase Windings

Once we get the expression of *mmf* for a single phase winding, it is not difficult to obtain the expressions of *mmf's* for three single phase windings placed 120º (electrical) apart and excited by balanced three phase currents given below, assuming *abc* sequence:

$$i_a = \sqrt{2}\, I_{rms} \cos(\omega t)$$

$$i_b = \sqrt{2}\, I_{rms} \cos(\omega t - 120)$$

$$i_c = \sqrt{2}\, I_{rms} \cos(\omega t + 120)$$

**For phase *a*,**

$$F_a = F_{m1}\cos(\omega t)\cos(\theta) + F_{m3}\cos(\omega t)\cos(3\theta) + \cdots + F_{mn}\cos(\omega t)\cos(n\theta)$$

$$F_{a1} = \frac{F_{m1}}{2}\cos(\theta - \omega t) + \frac{F_{m1}}{2}\cos(\theta + \omega t)$$

$$F_{a3} = \frac{F_{m3}}{2}\cos(3\theta - \omega t) + \frac{F_{m3}}{2}\cos(3\theta + \omega t)$$

$$\cdots$$

$$F_{an} = \frac{F_{mn}}{2}\cos(n\theta - \omega t) + \frac{F_{mn}}{2}\cos(n\theta + \omega t)$$

**For phase *b*,**

$$F_b = F_{m1}\cos(\omega t - 120)\cos(\theta - 120) + F_{m3}\cos(\omega t - 120)\cos 3(\theta - 120) + \cdots + F_{mn}\cos(\omega t - 120)\cos n(\theta - 120)$$

$$F_{b1} = \frac{F_{m1}}{2}\cos(\theta - \omega t) + \frac{F_{m1}}{2}\cos(\theta + \omega t - 240)$$

$$F_{b3} = \frac{F_{m3}}{2}\cos(3\theta - \omega t + 120) + \frac{F_{m3}}{2}\cos(3\theta + \omega t - 120)$$

$$\cdots$$

$$F_{bn} = \frac{F_{mn}}{2}\cos(n\theta - \omega t - 120(n-1)) + \frac{F_{mn}}{2}\cos(n\theta + \omega t - 120(n+1))$$

**For phase *c*,**

$$F_c = F_{m1}\cos(\omega t + 120)\cos(\theta + 120) + F_{m3}\cos(\omega t + 120)\cos 3(\theta + 120) + \ldots + F_{mn}\cos(\omega t + 120)\cos n(\theta + 120)$$

$$F_{c1} = \frac{F_{m1}}{2}\cos(\theta - \omega t) + \frac{F_{m1}}{2}\cos(\theta + \omega t + 240)$$

$$F_{c3} = \frac{F_{m3}}{2}\cos(3\theta - \omega t - 120) + \frac{F_{m3}}{2}\cos(3\theta + \omega t + 120)$$

$$\cdots$$

$$F_{cn} = \frac{F_{mn}}{2}\cos(n\theta - \omega t + 120(n-1)) + \frac{F_{mn}}{2}\cos(n\theta + \omega t + 120(n+1))$$

The resultant fundamental component of *mmf* generated by a three phase winding is obtained as:

$$F_1 = F_{a1} + F_{b1} + F_{c1}$$

$$= \frac{F_{m1}}{2}\cos(\theta - \omega t) + \frac{F_{m1}}{2}\cos(\theta + \omega t)$$

$$+ \frac{F_{m1}}{2}\cos(\theta - \omega t) + \frac{F_{m1}}{2}\cos(\theta + \omega t - 240)$$

$$+ \frac{F_{m1}}{2}\cos(\theta - \omega t) + \frac{F_{m1}}{2}\cos(\theta + \omega t + 240)$$

As we know that the sum of any three-phase component is zero, therefore

$$\frac{F_{m1}}{2}\cos(\theta + \omega t) + \frac{F_{m1}}{2}\cos(\theta + \omega t - 240) + \frac{F_{m1}}{2}\cos(\theta + \omega t + 240) = 0$$

$$F_1 = \frac{3\,F_{m1}}{2}\cos(\theta - \omega t)$$

As explained before, $F_{m1} = 0.9\,\frac{K_w N_{ph}\, I_{rms}}{P}$

$$F_1 = \frac{3}{2}\,0.9\,\frac{K_w N_{ph}\, I_{rms}}{P}\cos(\theta - \omega t) = 1.35\,\frac{K_w N_{ph}\, I_{rms}}{P}\cos(\theta - \omega t)$$

The above equation representing the total mmf of a 3-phase winding is shown in Fig. 51, at two different instants ($\omega t = 0$ and $\omega t = \pi/2$).

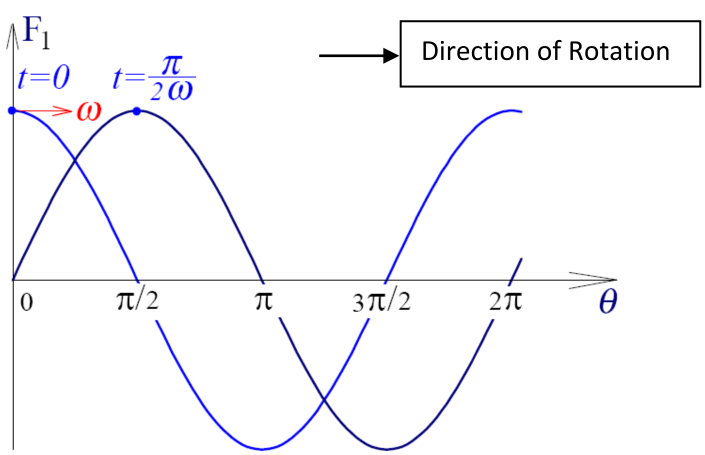



Fig. 51, Rotating of fundamental component of 3-phase winding MMF

It can be readily observed that $F_1$ is a rotating mmf with the synchronous speed in the $+\theta$ direction (a→b→c) with a constant magnitude $3F_m/2$.

Comparing with the relationship between the rotor speed and the frequency of the induced *emf* in a three phase winding obtained before, we can find that the rotor speed equals the rotating field speed for a given frequency. So this is why this speed is called the synchronous speed.

The resultant 3rd harmonic component of *mmf* generated by a three phase winding is obtained as:

$$F_3 = F_{a3} + F_{b3} + F_{c3}$$

$$= \frac{F_{m3}}{2}\cos(3\theta - \omega t) + \frac{F_{m3}}{2}\cos\ (3\theta + \omega t)$$

$$+ \frac{F_{m3}}{2}\cos(3\theta - \omega t + 120) + \frac{F_{m3}}{2}\cos(3\theta + \omega t - 120)$$

$$+ \frac{F_{m3}}{2}\cos(3\theta - \omega t - 120) + \frac{F_{m3}}{2}\cos(3\theta + \omega t + 120)$$

Since the sum of any 3-phase components is zero, then

$$\frac{F_{m3}}{2}\cos(3\theta - \omega t) + \frac{F_{m3}}{2}\cos(3\theta - \omega t + 120) + \frac{F_{m3}}{2}\cos(3\theta - \omega t - 120) = 0$$

and

$$\frac{F_{m3}}{2}\cos(3\theta + \omega t) + \frac{F_{m3}}{2}\cos(3\theta + \omega t - 120) + \frac{F_{m3}}{2}\cos(3\theta + \omega t + 120) = 0$$

Then

$$F_3 = 0$$

For the 5th harmonics component of MMF can be obtained from:

$$F_5 = F_{a5} + F_{b5} + F_{c5}$$

$$= \frac{F_{m5}}{2}\cos(5\theta - \omega t) + \frac{F_{m5}}{2}\cos\ (5\theta + \omega t)$$

$$+ \frac{F_{m5}}{2}\cos(5\theta - \omega t - 120) + \frac{F_{m5}}{2}\cos(5\theta + \omega t)$$

$$+ \frac{F_{m5}}{2}\cos(5\theta - \omega t + 120) + \frac{F_{m5}}{2}\cos(5\theta + \omega t)$$

Since the sum of any 3-phase components is zero, then

$$\frac{F_{m5}}{2}\cos(5\theta-\omega t)+\frac{F_{m5}}{2}\cos(5\theta-\omega t-120)+\frac{F_{m5}}{2}\cos(5\theta-\omega t+120)=0$$

Therefore,

$$F_5=\frac{3\,F_{m5}}{2}\cos(5\theta+\omega t)=1.35\,\frac{K_{w5}N_{ph}\,I_{rms}}{5P}\cos(5\theta+\omega t)$$

Which is rotating wave in a direction opposite to the fundamental at a speed of (ω/5)

By the same way, the 7$^{th}$ harmonic component of MMF is

$$F_7=\frac{3\,F_{m7}}{2}\cos(7\theta-\omega t)=1.35\,\frac{K_{w}N_{ph}\,I_{rms}}{7P}\cos(7\theta-\omega t)$$

Which is rotating wave in the same direction of the fundamental at a speed of (ω/7)

Again, the 9$^{th}$ harmonic component of MMF is zero

$$F_9=0$$

Generally, for the $n^{th}$ harmonic component of MMF

$$F_n=1.35\,\frac{K_{wn}N_{ph}\,I_{rms}}{nP}\cos(n\theta\pm\omega t)$$

if we use the +ve sign, this mean the direction of rotation is the same as fundamental. But if we use the -ve sign, this mean the direction of rotation is opposite to fundamental.

As we know, the third harmonic and its multiples components of MMF are zero, therefore, we can assign the index *m* such that

$$n=\;6m\pm1$$

where *m* is integer = 0,1,2,3,...

at *m* = 0, we get *n* = 1 which is the fundamental

at *m* = 1, we get *n* = 5 (-ve sign, opposite direction) OR *n* = 7 (+ve sign, same direction)

at *m* =2, we get *n* = 11 (-ve sign, opposite direction) OR *n* = 13 (+ve sign, same direction)

at *m* =3, we get *n* = 17 (-ve sign, opposite direction) OR *n* = 19 (+ve sign, same direction)

. . . etc as shown in Fig. 52.

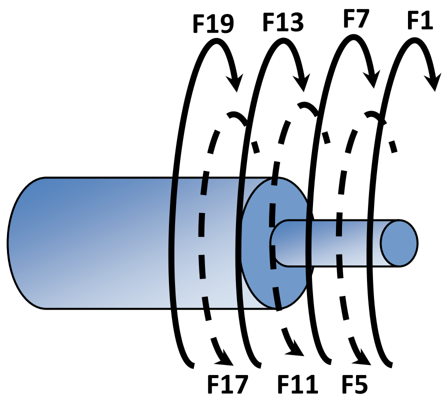



Fig. 52, Direction of rotation for fundamental component and harmonics of MMF

The above analytical derivation can also be done graphically by using adding the *mmf* vectors of three phases, as illustrated in the diagrams shown in Fig. 53.

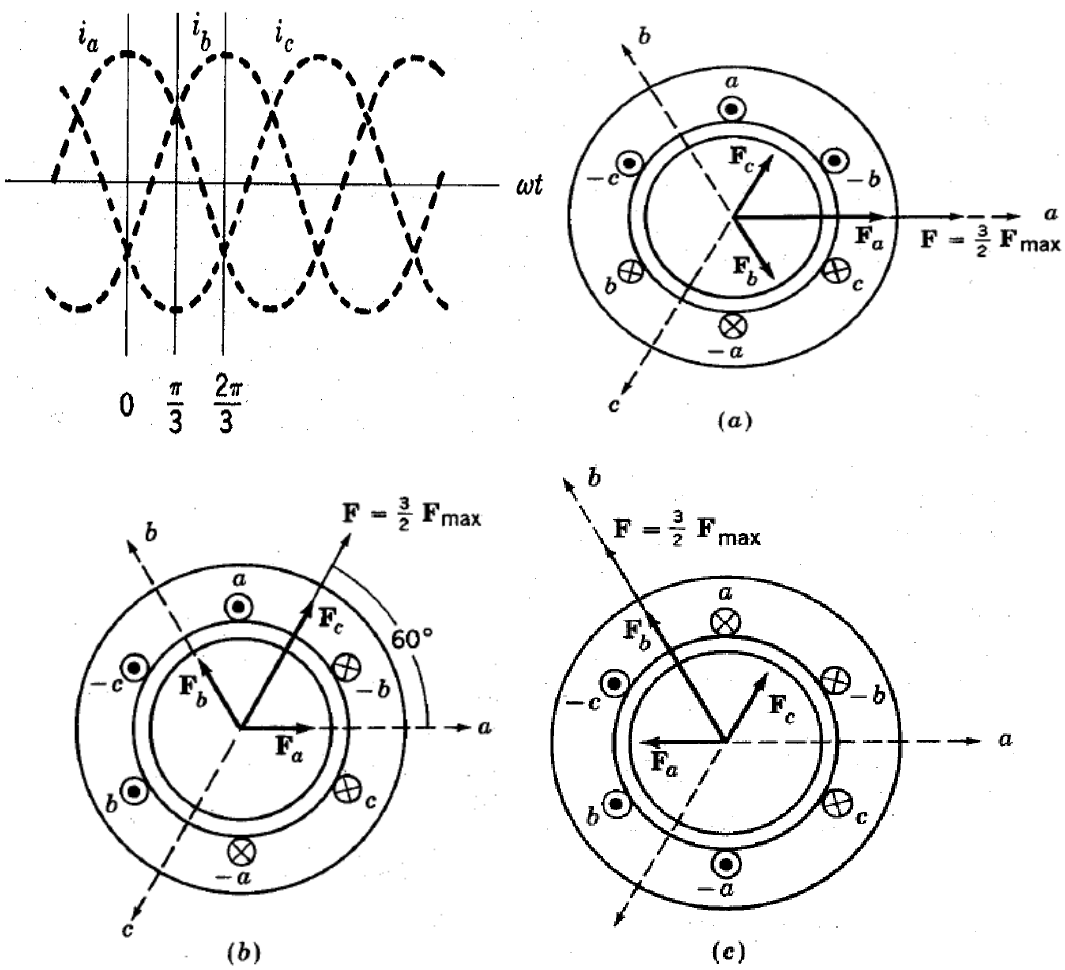



Fig. 53, graphical proof of rotating mmf of the 3-phase winding

* When $\omega t=0$, phase *a* current is maximum and the *mmf* vector with a magnitude $F_m$ of phase *a* is on the magnetic axis of phase a, while the *mmf's* of phases *b* and *c* are both of magnitude $F_m/2$ and in the opposite directions of their magnetic axes since the currents of these two phases are both $-I_m/2$. Therefore, the resultant *mmf* $F_1=3F_m/2$ is on the magnetic axis of phase *a* as shown in Fig. 53-a.

* When $\omega t = \pi/3$, $i_c=-I_m$ and $i_a=i_b=I_m/2$. The resultant *mmf* $F_1=3F_m/2$ is on the axis of phase *c* but in the opposite direction as shown in Fig. 53-b.

* When $\omega t=2\pi/3$, $i_b=I_m$ and $i_a=i_c=-I_m/2$. Hence the resultant *mmf* $F_1=3F_m/2$ is in the positive direction of the magnetic axis of phase *b* as shown in Fig. 53-c.

In general, the resultant *mmf* is of a constant magnitude of $3F_m/2$ and will be in the positive direction of the magnetic axis of a phase winding when the current in that phase winding reaches positive maximum. The speed of the rotating *mmf* equals the angular frequency in electrical rad/s.

**In the case of a synchronous generator**, three balanced *emf's* of frequency $f=Pn/60$ Hz are induced in the three phase windings when the rotor is driven by a prime mover rotating at a speed *n*. If the three phase stator circuit is closed by a balanced three phase electrical load, balanced three phase currents of frequency *f* will flow in the stator circuit, and these currents will generate a rotating magnetic field of a speed $n_f = 60f/P = n$.

**In the case of a synchronous motor,** the stator winding is supplied by a balanced three phase power supply of frequency *f*, the balanced three phase currents in the winding will generate a rotating magnetic field of speed $n_f = 60f/P$. This rotating magnetic field will drag the magnetized rotor, which is essential a magnet, to rotate at the same speed $n=n_f$. On the other hand, this rotating rotor will also generate balanced three phase *emf's* of frequency *f* in the stator winding, which would balance with the applied terminal voltage.

## 7. Effect of Harmonics on chording and Distribution Factors

As said before, the square wave represents the mmf waveform shown in Fig. 47, is resolved to a fundamental component ($F_1$) and odd harmonics ($F_3$ & $F_5$ & $F_7$ and so on).

### 7.1 Chording Factor for odd harmonic

If the coil pitch is chorded by γ,

For 3rd harmonic the chording angle is 3γ

$$K_{c3} = \cos\left(\frac{3\gamma}{2}\right)$$

For 5th harmonic the chording angle is 5γ

$$K_{c5} = \cos\left(\frac{5\gamma}{2}\right)$$

For 7th harmonic the chording angle is 7γ

$$K_{c7} = \cos\left(\frac{7\gamma}{2}\right)$$

### 7.2 Distribution Factor for odd harmonic

For a coil group of Q coils per pole per phase and a slot angle α,

For 3rd harmonic

$$K_{d3} = \frac{sin\left(\frac{3Q\alpha}{2}\right)}{Q\ \sin\left(\frac{3\alpha}{2}\right)}$$

For 5th harmonic

$$K_{d5} = \frac{sin\left(\frac{5Q\alpha}{2}\right)}{Q\ \sin\left(\frac{5\alpha}{2}\right)}$$

For 7th harmonic

$$K_{d7} = \frac{sin\left(\frac{7Q\alpha}{2}\right)}{Q\ \sin\left(\frac{7\alpha}{2}\right)}$$

## 7.3 Third Harmonic and its integer multiples

The 3rd harmonic and all of its integer multiples (collectively called *triplen* harmonics) generated by 120$^o$ phase-shifted fundamental waveforms are actually in phase with each other. In a 60 Hz three-phase power system, where phases A, B, and C are 120$^o$ apart, the third-harmonic multiples of those frequencies (180 Hz) fall perfectly into phase with each other. This can be thought of in graphical terms as shown in Fig. 54.

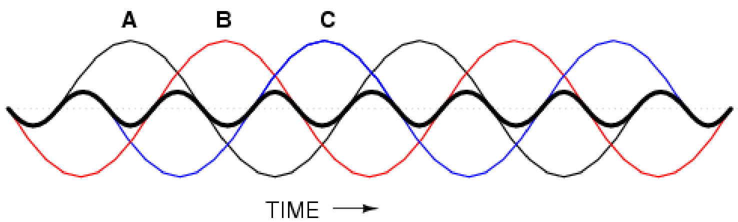



Fig. 54, Triplen harmonics

*Harmonic currents of Phases A, B, C all coincide, that is, no rotation.*

*Phase sequence =* ***A-B-C***

| | | | |
|---|---|---|---|
| Fundamental | **A**<br>0° | **B**<br>120° | **C**<br>240° |
| 3rd harmonic | **A'**<br>3 x 0°<br>(0°) | **B'**<br>3 x 120°<br>(360° = 0°) | **C'**<br>3 x 240°<br>(720° = 0°) |

If we extend the mathematical table to include higher odd-numbered harmonics, we will notice an interesting pattern develop with regard to the rotation or sequence of the harmonic frequencies:

| | | | | |
|---|---|---|---|---|
| Fundamental | **A**<br>0° | **B**<br>120° | **C**<br>240° | **A-B-C** |
| 3rd harmonic | **A'**<br>3 x 0°<br>(0°) | **B'**<br>3 x 120°<br>(360° = 0°) | **C'**<br>3 x 240°<br>(720° = 0°) | *no rotation* |
| 5th harmonic | **A''**<br>5 x 0°<br>(0°) | **B''**<br>5 x 120°<br>(600° = 720° - 120°)<br>(-120°) | **C''**<br>5 x 240°<br>(1200° = 1440° - 240°)<br>(-240°) | **C-B-A** |
| 7th harmonic | **A'''**<br>7 x 0°<br>(0°) | **B'''**<br>7 x 120°<br>(840° = 720° + 120°)<br>(120°) | **C'''**<br>7 x 240°<br>(1680° = 1440° + 240°)<br>(240°) | **A-B-C** |
| 9th harmonic | **A''''**<br>9 x 0°<br>(0°) | **B''''**<br>9 x 120°<br>(1080° = 0°) | **C''''**<br>9 x 240°<br>(2160° = 0°) | *no rotation* |

Harmonics such as the 7th, which "rotate" with the same sequence as the fundamental, are called *positive sequence*. Harmonics such as the 5th, which "rotate" in the opposite sequence as the fundamental, are called *negative sequence*. Triplen harmonics (3rd and 9th shown in table above) which don't "rotate" at all because they're in phase with each other, are called *zero sequence*.

This pattern of positive-zero-negative-positive continues indefinitely for all odd-numbered harmonics, lending itself to expression in a table like this:

*Rotation sequences according to harmonic number*

| | | | | | |
|---|---|---|---|---|---|
| + | 1st | 7th | 13th | 19th | ← Rotates with fundamental |
| 0 | 3rd | 9th | 15th | 21st | ← Does not rotate |
| - | 5th | 11th | 17th | 23rd | ← Rotates against fundamental |

Sequence especially matters when we're dealing with AC motors, since the mechanical rotation of the rotor depends on the torque produced by the sequential "rotation" of the applied 3-phase power. Positive-sequence frequencies work to push the rotor in the proper direction, whereas negative-sequence frequencies actually work *against* the direction of the rotor's rotation. Zero-sequence frequencies neither contribute to nor detract from the rotor's torque. An excess of negative-sequence harmonics (5th, 11th, 17th, and/or 23rd) in the power supplied to a three-phase AC motor will result in a degradation of performance and possible overheating. Since the higher-order harmonics tend to be attenuated more by system inductances and magnetic core losses, and generally originate with less amplitude anyway, the primary harmonic of concern is the 5th, which is 300 Hz in 60 Hz power systems and 250 Hz in 50 Hz power systems.

**<u>Example (19):</u>**

A 10-pole, 50-Hz, 600 r.p.m. alternator has flux density distribution given by the following expression:

$$B = \sin\theta + 0.4\sin 3\theta + 0.2\sin 5\theta$$

The alternator has 180 slots wound with 2-layer 3-turn coils having a span of 15 slots. The coils are connected in 60° groups. If armature diameter is = 1.2 m and core length = 0.4 m, calculate the r.m.s. phase and line voltages, if the machine is star-connected.

First, calculate the fundamental magnetic flux $\Phi_1$ = Average flux density * area of pole pitch

Area of pole pitch ($A_1$)= $\pi$ D L/2P = $\pi$ (1.2)(0.4)/10 = 0.1508 $m^2$

Average flux density of the fundamental component $B_{1\,av} = \frac{2}{\pi} \times B_{1\,max}$ = 0.637 Tesla

$\Phi_1$ = 0.1508 * 0.637 = 0.09606 Wb

Considering that, the area of 3rd harmonic is one-third the fundamental area ($A_1$)

By the same way $\Phi_3$ = (0.1508/3) * 0.637*0.4 = 0.012808 Wb

By the same way $\Phi_5$ = (0.1508/5) * 0.637*0.2 = 0.0038424 Wb

Since the winding is double layer, then there are 2 coil sides per slot, this mean each slot carry one coil that has 3 turns.

Total number of turns = 180 * 3 = 540

$N_{ph}$ = 540/3 = 180 turns/phase

Pole pitch = S / (2P) = 180 / 10 = 18 and the slot angle $\alpha$ = 180/18 = 10 elec. degrees

Coil pitch = 15  Also Q = S/(6P) = 180/(6*5) = 6

Then there is chording by 3 slots, therefore $\gamma$ = 3*10 = 30

$$K_{c1} = \cos\left(\frac{\gamma}{2}\right) = \cos\left(\frac{30}{2}\right) = 0.966$$

$$K_{c3} = \cos\left(\frac{3\gamma}{2}\right) = \cos\left(\frac{90}{2}\right) = 0.7071$$

$$K_{c5} = \cos\left(\frac{5\gamma}{2}\right) = \cos\left(\frac{150}{2}\right) = 0.25882$$

***The distribution factors***

$$K_{d1} = \frac{sin\left(\frac{Q\alpha}{2}\right)}{Q\ \sin\left(\frac{\alpha}{2}\right)} = \frac{sin\left(\frac{6\times 10}{2}\right)}{6\ sin\left(\frac{10}{2}\right)} = 0.95614$$

$$K_{d3} = \frac{sin\left(\frac{3Q\alpha}{2}\right)}{Q\ \sin\left(\frac{3\alpha}{2}\right)} = \frac{sin\left(\frac{180}{2}\right)}{6\ sin\left(\frac{30}{2}\right)} = 0.64395$$

$$K_{d5} = \frac{sin\left(\frac{5Q\alpha}{2}\right)}{Q\ \sin\left(\frac{5\alpha}{2}\right)} = \frac{sin\left(\frac{300}{2}\right)}{6\ sin\left(\frac{50}{2}\right)} = 0.1972$$

$K_{w1} = K_{c1} * K_{d1} = 0.966 * 0.95614 = 0.92363$

$K_{w3} = K_{c3} * K_{d3} = 0.7071 * 0.64395 = 0.455337$

$K_{w5} = K_{c5} * K_{d5} = 0.25882 * 0.1972 = 0.05104$

***Frequencies***

$f_1 = 50$ Hz $\quad$ $f_3 = 3*50 = 150$ Hz $\quad$ $f_5 = 5*50 = 250$ Hz

***Phase voltages***

$E_1 = 4.44\ f_1\ \Phi_1\ N_{ph}\ K_{w1}$

$= 4.44 * 50 * 0.09606 * 180 * 0.92363 = 3545.407$ volt

$E_3 = 4.44\ f_3\ \Phi_3\ N_{ph}\ K_{w3}$

$= 4.44 * 150 * 0.012808 * 180 * 0.455337 = 699.1349$ volt

$E_5 = 4.44\ f_5\ \Phi_5\ N_{ph}\ K_{w5}$

$= 4.44 * 250 * 0.0038424 * 180 * 0.1972 = 151.392712$ volt

Total voltage $E = \sqrt{E_1^2 + E_3^2 + E_5^2}$

$E = \sqrt{3545.407^2 + 699.1349^2 + 151.392712^2} = 3616.85225$ volt

As the machine is ***star connected***, the third harmonic of the line voltage and its odd multiplications (e.g. 9th, 15th, 21th, … etc) become zero

Line voltage = $\sqrt{3}\sqrt{3545.407^2 + 151.392712^2} = 6146.421$ volt

**Example (20):**

*A 3-phase alternator has generated e.m.f. per phase of 230 V with 10 percent third harmonic and 6 percent fifth harmonic content. Calculate the r.m.s. line voltage for **(a)** star connection **(b)** delta-connection. Find also the circulating current in delta connection if the reactance per phase of the machine at 50-Hz is 10 Ω.*

$E_1 = 230$ V ; $E_3 = 0.1 \times 230 = 23$ V ; $E_5 = 0.06 \times 230 = 13.8$ V

| In Case of Y-connected | In case of Delta-connected |
|---|---|
| $E_{ph} = \sqrt{230^2 + 23^2 + 13.8^2} = 231.56$ V | $E_{ph} = \sqrt{230^2 + 23^2 + 13.8^2} = 231.56$ V |
| $E_{Line} = \sqrt{3}\sqrt{230^2 + 13.8^2} = 399.1$ V | $E_L = E_{ph}$ |

**8. MMF Waveforms**

Consider the machine given in single layer concentric winding (section 4.1.1) where S=24 & 2P = 4 & Q = 2

when the current entering the slot is represented by "X" and produce an MMF in the upward direction. But when leaving the slot is represented by "o" and produce an MMF in the down ward direction.

the overall MMF is as shown in Fig. 55.

**Example (21)**:

Consider a 3-phase, 2-poles, double layer windings with number of slots per pole per phase equal 2. It is required to draw the total MMF waveform for the instant when the current in phase C is zero and the currents in phases A and B are equal and of opposite sign. The coil pitch is 5.

Q= S/6P = 2 therefore S = 12 τ=S/2P = 12/2 = 6

Since the coil pitch is 5, the winding is chorded by one slot. The total MMF is shown in Fig. 56.

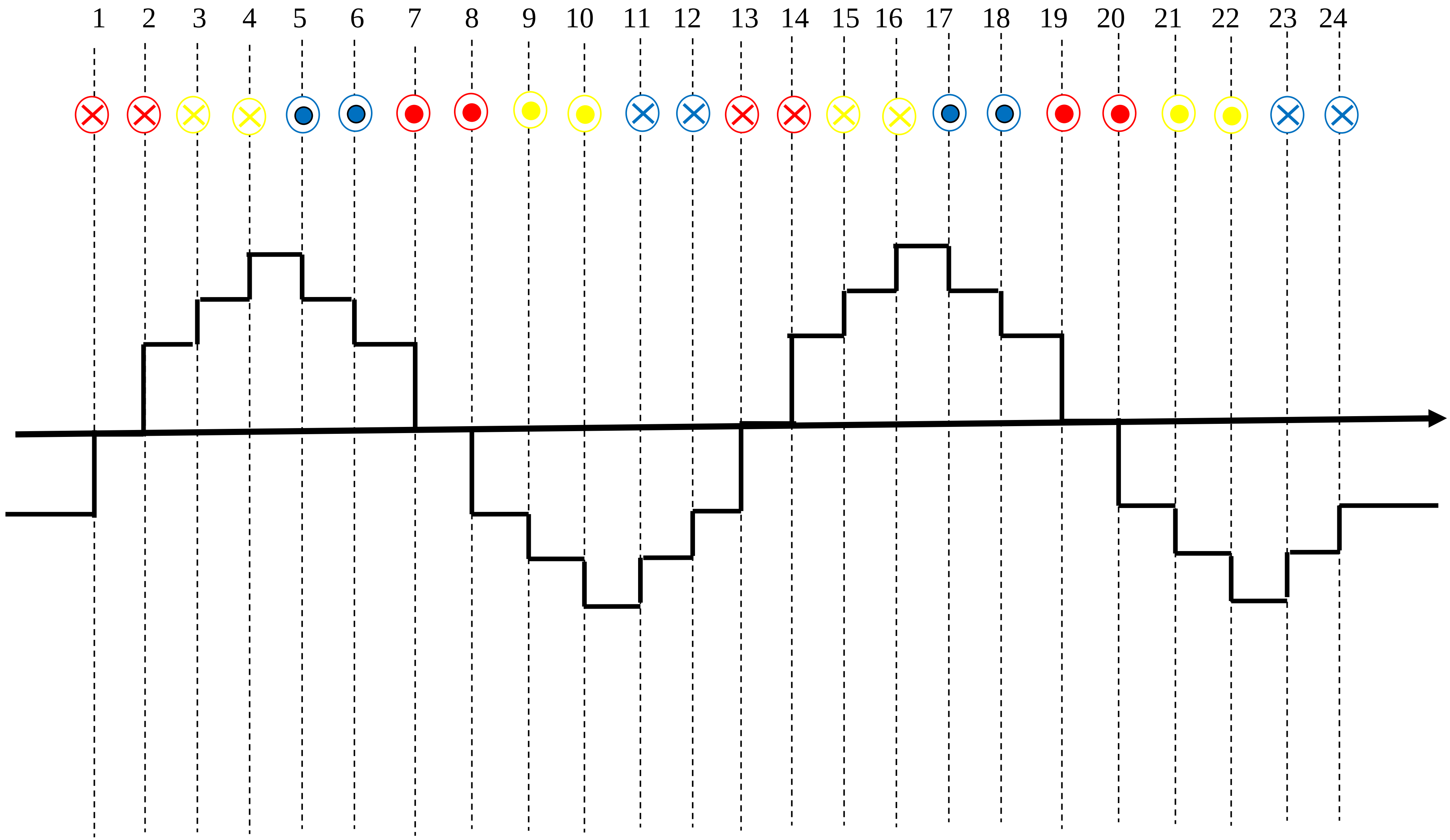



Fig. 55 MMF waveform of 3-phase single layer winding

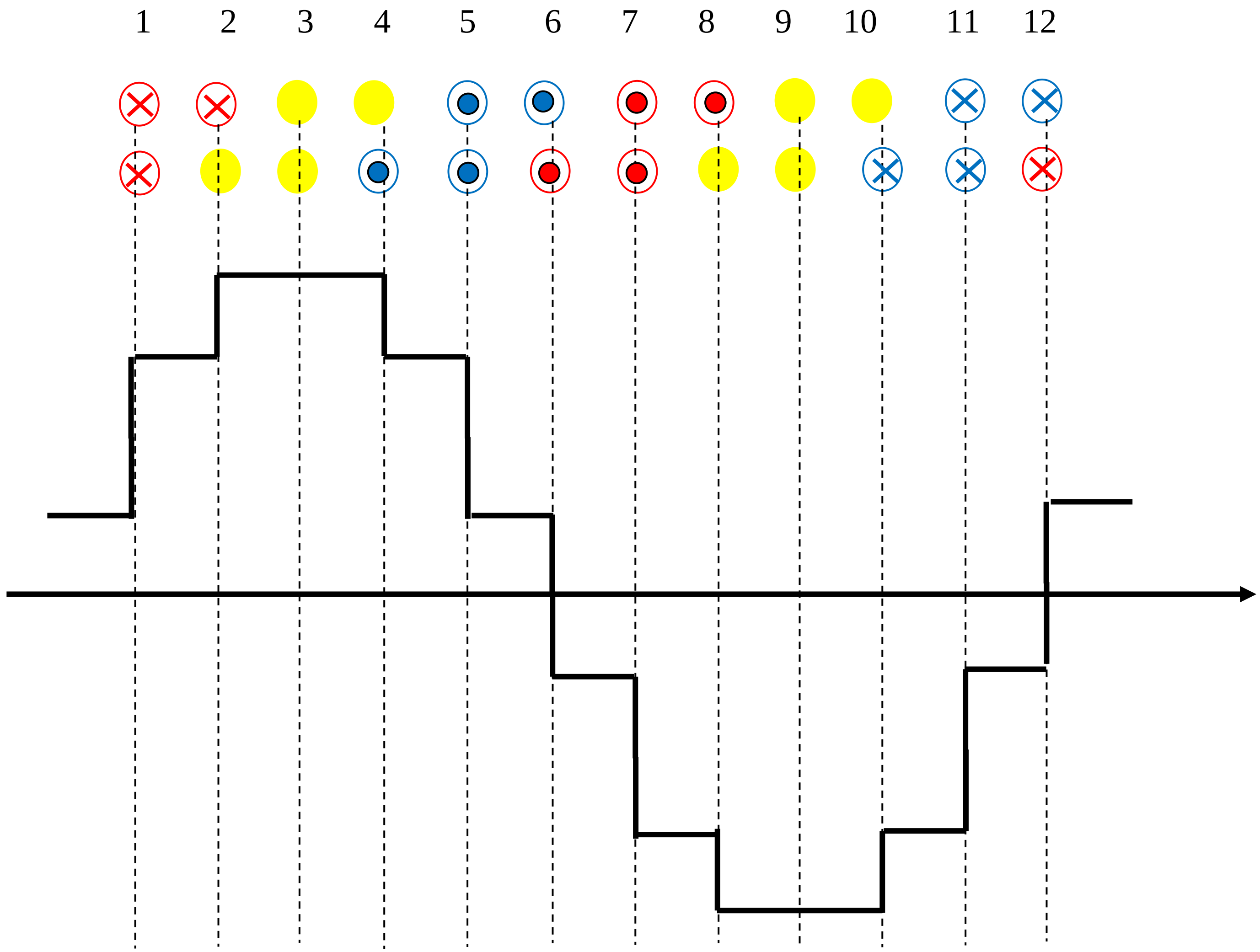



Fig. 56 MMF waveform of 3-phase double layer winding

**Example (22)**

A three-phase, 375 kVA, 50 Hz, star-connected generator running at 300 rpm, is designed to develop 4160 V between terminals. The double layer winding consists of 150 coils, each coil having 9 turns and a pitch of six slots. Determine the stator MMF in amper-turn per pole (AT/P) when the machine is delivering its rated current.

Solution

since the winding is double layer, then the number of slots = the number of coils

S = 150

n = 60*f*/P → P = 60 *f* / n = 60×50/300 = 10

Pole pitch, τ=S/2P = 150/20 = 7.5

but the coil pitch is 6, this mean there is chording by 1.5 slots

the slot angle (α) = 180/ τ = 180/7.5 = 24 electrical degrees

the chording angle (γ) = 1.5 × 24 = 36 electrical degrees

coil group (Q) = S/6P = 150 / 60 = 2.5 (fractional slot winding)

number of turns/phase ($N_{ph}$) = coils × turns per coil / number of phases = 150 × 9 /3 = 450 turns Or

number of turns/phase ($N_{ph}$) = Q × 2P × turns per coil = 2.5 × 20 × 9 = 450 turns

Winding factor

$$K_{c1} = \cos\left(\frac{\gamma}{2}\right) = \cos\left(\frac{36}{2}\right) = 0.951$$

Distribution factor

$$K_{d1} = \frac{sin\left(\frac{Q\alpha}{2}\right)}{Q\ \sin\left(\frac{\alpha}{2}\right)} = \frac{sin\left(\frac{2.5 \times 24}{2}\right)}{2.5\ sin\left(\frac{24}{2}\right)} = 0.962$$

Winding factor = $K_c \times K_d$ = 0.951 × 0.962 = 0.915

from power equation

$P = \sqrt{3}\ V_L\ I_L$

$375000 = \sqrt{3} \times 4160 \times I_L \rightarrow I_L = 52.046$ A

The MMF

$$F_1 = 1.35\ \frac{K_w N_{ph}\ I_{rms}}{P} = 1.35\ \frac{0.915 \times 450 \times 52.046}{10} = 2893.042\ AT/P$$

**Example (23)**

the no-load flux density distribution of a 3-phase, 14-poles, 50 Hz, Y-connected synchronous machine is represented by:

B = 100 sin X + 14 sin 3X + 20 sin 5X + sin 7X

The machine has a stator with 84 slots and double layer windings with 18 conductors per slot and a coil pitch of 5 slots. If the fundamental flux per pole is 0.0212 Wb, determine:

a) E3/E1 b) E5/E1 c) E7/E1 d) the rms phase and line voltages

Solution

the max. flux density of the fundamental component ($B_{1\ max}$) = 100 Tesla

the max. flux density of the 3rd harmonic component ($B_{3\ max}$) = 14 Tesla

the max. flux density of the 5th harmonic component ($B_{5\ max}$) = 20 Tesla

the max. flux density of the 7th harmonic component ($B_{7\ max}$) = 1 Tesla

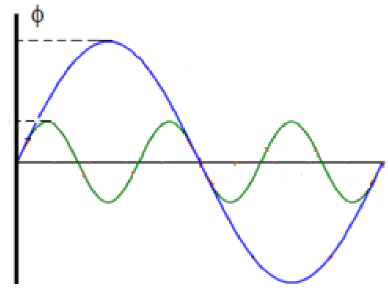
***Frequencies***

$f_1$ = 50 Hz $f_3$ = 3×50 = 150 Hz $f_5$ = 5×50 = 250 Hz $f_7$ = 7×50 = 350 Hz

Total number of conductors = 84 × 18 = 1512 conductor

Total number of turns = 1512 / 2 = 756

Number of turns per phase = 756 / 3 = 252 turns/phase




as shown from figure above, the pole pitch (area) of the fundamental component is 3 times larger than the pole pitch (area) of the 3rd harmonic

this mean $A_1 = 3\ A_3$

$$\frac{A_1}{A_3} = 3 \ \ or\ in\ general\ \frac{A_1}{A_n} = n$$

Also, $\phi_1 = B_1 \times A_1$ and $\phi_3 = B_3 \times A_3$

$$\frac{\phi_1}{\phi_3} = \frac{B_1}{B_3}\frac{A_1}{A_3} = \frac{B_1}{B_3} \times 3$$

In general:

$$\frac{\phi_1}{\phi_n} = \frac{B_1}{B_n} \times n$$

$$\phi_n = \phi_1 \frac{B_n}{B_1} \times \frac{1}{n}$$

$$\phi_3 = 0.0212 \times \frac{14}{100} \times \frac{1}{3} = 9.893 \times 10^{-4}\ Wb$$

$$\phi_5 = 0.0212 \times \frac{20}{100} \times \frac{1}{5} = 8.48 \times 10^{-4}\ Wb$$

$$\phi_7 = 0.0212 \times \frac{1}{100} \times \frac{1}{7} = 3.029 \times 10^{-5}\ Wb$$

Pole pitch, $\tau = S/2P = 84/14 = 6$, but the coil pitch is 5, this mean there is chording by one slot.

the slot angle ($\alpha$) = 180/ $\tau$ = 180/6 = 30 electrical degrees

the chording angle ($\gamma$) = 1.0 × 30 = 30 electrical degrees

coil group (Q) = S/6P = 84 / 42 = 2.0

***The chording factors***

$$K_{c1} = \cos\left(\frac{\gamma}{2}\right) = \cos\left(\frac{30}{2}\right) = 0.966$$

$$K_{c3} = \cos\left(\frac{3\gamma}{2}\right) = \cos\left(\frac{90}{2}\right) = 0.7071$$

$$K_{c5} = \cos\left(\frac{5\gamma}{2}\right) = \cos\left(\frac{150}{2}\right) = 0.25882$$

$$K_{c7} = \cos\left(\frac{7\gamma}{2}\right) = \cos\left(\frac{210}{2}\right) = -0.25882$$

***The distribution factors***

$$K_{d1} = \frac{sin\left(\frac{Q\alpha}{2}\right)}{Q\ \sin\left(\frac{\alpha}{2}\right)} = \frac{sin\left(\frac{2\times 30}{2}\right)}{2\ sin\left(\frac{30}{2}\right)} = 0.966$$

$$K_{d3} = \frac{sin\left(\frac{3Q\alpha}{2}\right)}{Q\ \sin\left(\frac{3\alpha}{2}\right)} = \frac{sin\left(\frac{180}{2}\right)}{2\ sin\left(\frac{90}{2}\right)} = 0.7071$$

$$K_{d5} = \frac{sin\left(\frac{5Q\alpha}{2}\right)}{Q\ \sin\left(\frac{5\alpha}{2}\right)} = \frac{sin\left(\frac{300}{2}\right)}{2\ sin\left(\frac{150}{2}\right)} = 0.25882$$

$$K_{d7} = \frac{sin\left(\frac{7Q\alpha}{2}\right)}{Q\ \sin\left(\frac{7\alpha}{2}\right)} = \frac{sin\left(\frac{420}{2}\right)}{2\ sin\left(\frac{210}{2}\right)} = -0.25882$$

$K_{w1} = K_{c1} * K_{d1} = 0.966 * 0.966 = 0.933$

$K_{w3} = K_{c3} * K_{d3} = 0.7071 * 0.7071 = 0.5$

$K_{w5} = K_{c5} * K_{d5} = 0.25882 * 0.25882 = 0.067$

$K_{w7} = K_{c7} * K_{d7} = -0.25882 * - 0.25882 = 0.067$

$E_1 = 4.44\ f_1\ \Phi_1\ N_{ph}\ K_{w1}$

$= 4.44 * 50 * 0.0212 * 252 * 0.933 = 1106.545$ volt

$E_3 = 4.44\ f_3\ \Phi_3\ N_{ph}\ K_{w3}$

$= 4.44 * 150 * 9.893\times10^{-4} * 252 * 0.5 = 83.018$ volt

$E_5 = 4.44\ f_5\ \Phi_5\ N_{ph}\ K_{w5}$

$= 4.44 * 250 * 8.48\times10^{-4} * 252 * 0.067 = 15.892$ volt

$E_7 = 4.44\ f_7\ \Phi_7\ N_{ph}\ K_{w7}$

$= 4.44 * 350 * 3.029\times10^{-5} * 252 * 0.067 = 0.795$ volt

$E_3 / E_1 = 83.018 / 1106.545 = 0.075$

$E_5 / E_1 = 15.892 / 1106.545 = 0.014$

$E_7 / E_1 = 0.795 / 1106.545 = 7.1845\times10^{-4}$

Total phase voltage $E = \sqrt{E_1^2 + E_3^2 + E_5^2 + E_7^2}$

$E_{ph} = \sqrt{1106.545^2 + 83.018^2 + 15.892^2 + 0.795^2} = 1109.7689$ volt

As the machine is ***star connected***, the third harmonic of the line voltage and its odd multiplications (e.g. $9^{th}$, $15^{th}$, $21^{th}$, … etc) become zero

Total line voltage $E = \sqrt{3}\sqrt{E_1^2 + E_5^2 + E_7^2}$

$E_L = \sqrt{3}\sqrt{1106.545^2 + 15.892^2 + 0.795^2} = 1915.058$ volt

**Example (24)**

1000 kVA, 8-pole, 50-Hz, Y-connected alternator has an average air-gap flux distribution as: $\Phi = 0.9 \sin\theta + 0.3 \sin 3\theta + 0.1 \sin 5\theta + 0.05 \sin 7\theta + 0.01 \sin 9\theta$

The alternator has 120 slots wound with double-layer 7-turn coils having a span from slot 1 to slot 13.

a) Calculate the rated speed in r.p.m.

b) Calculate the rated phase and line voltages.

c) Calculate the total MMF produced by the 3-ph armature winding.

a) Speed = 60×$f$ / P = 60 × 50 / 4 = 750 rpm

b) $\Phi_1$ = 0.9 Wb/pole $f_1$ = 50 Hz

$\Phi_3$ = 0.3 Wb/pole $f_3$ = 150 Hz

$\Phi_5$ = 0.1 Wb/pole $f_5$ = 250 Hz

$\Phi_7$ = 0.05 Wb/pole $f_7$ = 350 Hz

$\Phi_9$ = 0.01 Wb/pole $f_9$ = 450 Hz

Since the winding is double layer, then there are 2 coil sides per slot, this mean each slot carry one coil that has 7 turns.

Total number of turns = 120 * 7 = 840 turns and $N_{ph}$ = 840/3 = 280 turns/phase

Pole pitch = S / (2P) = 120 / 8 = 15 and the slot angle α = 180/15 = 12 elec. degrees

Coil pitch = 12. Then there is chording by 3 slots, therefore γ = 3×12 = 36

$$K_{c1} = \cos\left(\frac{\gamma}{2}\right) = \cos\left(\frac{36}{2}\right) = 0.951$$

$$K_{c3} = \cos\left(\frac{3\gamma}{2}\right) = \cos\left(\frac{108}{2}\right) = 0.5878$$

$$K_{c5} = \cos\left(\frac{5\gamma}{2}\right) = \cos\left(\frac{180}{2}\right) = 0.0$$

$$K_{c7} = \cos\left(\frac{7\gamma}{2}\right) = \cos\left(\frac{252}{2}\right) = -0.5878$$

$$K_{c9} = \cos\left(\frac{9\gamma}{2}\right) = \cos\left(\frac{324}{2}\right) = -0.951$$

Also Q = S/(6P) = 120/(6×4) = 5 Then ***The distribution factors***

$$K_{d1} = \frac{sin\left(\frac{Q\alpha}{2}\right)}{Q\ \sin\left(\frac{\alpha}{2}\right)} = \frac{sin\left(\frac{5\times 12}{2}\right)}{5\ sin\left(\frac{12}{2}\right)} = 0.9567$$

$$K_{d3} = \frac{sin\left(\frac{3Q\alpha}{2}\right)}{Q\ \sin\left(\frac{3\alpha}{2}\right)} = \frac{sin\left(\frac{180}{2}\right)}{5\ sin\left(\frac{36}{2}\right)} = 0.6472$$

$$K_{d5} = \frac{sin\left(\frac{5Q\alpha}{2}\right)}{Q\ \sin\left(\frac{5\alpha}{2}\right)} = \frac{sin\left(\frac{300}{2}\right)}{5\ sin\left(\frac{60}{2}\right)} = 0.2$$

$$K_{d7} = \frac{sin\left(\frac{7Q\alpha}{2}\right)}{Q\ \sin\left(\frac{7\alpha}{2}\right)} = \frac{sin\left(\frac{420}{2}\right)}{5\ sin\left(\frac{84}{2}\right)} = -0.1494$$

$$K_{d9} = \frac{sin\left(\frac{9Q\alpha}{2}\right)}{Q\ \sin\left(\frac{9\alpha}{2}\right)} = \frac{sin\left(\frac{540}{2}\right)}{5\ sin\left(\frac{108}{2}\right)} = -0.2472$$

$K_{w1} = K_{c1} * K_{d1} = 0.951 * 0.9567 = 0.9098217$

$K_{w3} = K_{c3} * K_{d3} = 0.5878 * 0.6472 = 0.3804242$

$K_{w5} = K_{c5} * K_{d5} = 0.0 * 0.\ 2 = 0.0$

$K_{w7} = K_{c7} * K_{d7} = -0.5878 * -0.1494 = 0.08782$

$K_{w9} = K_{c9} * K_{d9} = -0.951 * -0.2472 = 0.2351$

***Phase voltage***

$E_1 = 4.44\ f_1\ \Phi_1\ N_{ph}\ K_{w1}$

$= 4.44 * 50 * 0.9 * 280 * 0.9098217 = 50899.06518$ V

$E_3 = 4.44\ f_3\ \Phi_3\ N_{ph}\ K_{w3}$

$= 4.44 * 150 * 0.3 * 280 * 0.3804242 = 21282.45144$ V

$E_5 = 4.44\ f_5\ \Phi_5\ N_{ph}\ K_{w5}$

$= 4.44 * 250 * 0.1 * 280 * 0.0 = 0$ V

$E_7 = 4.44\ f_7\ \Phi_7\ N_{ph}\ K_{w7}$

= 4.44 * 350 * 0.05 * 280 * 0.08782 = 1910.61192 V

$E_9 = 4.44\, f_9\, \Phi_9\, N_{ph}\, K_{w9}$

= 4.44 * 450 * 0.01 * 280 * 0.2351 = 1315.2434 V

$E_{ph} = \sqrt{E_1^2 + E_3^2 + E_5^2 + E_7^2 + E_9^2}$

$E = \sqrt{50899.06518^2 + 21282.45144^2 + 0^2 + 1910.61192^2 + 1315.2434^2} = 55218.09375$ volt

As the machine is ***star connected***, the third harmonic of the line voltage and its odd multiplications (e.g. 9th, 15th, 21th, … etc) become zero

Line voltage = $\sqrt{3}\sqrt{50899.06518^2 + 1910.61192^2}$ = 88221.8557 volt

c) The armature current $I_a = \frac{1000\ kVA}{\sqrt{3}\times 88221.8557} = 6.5443\ A$

Armature reaction ($F_a$)

$$F_n = 1.35\ \frac{K_{wn} N_{ph}\, I_a}{nP}$$

$$F_1 = 1.35\ \frac{0.9098217 \times 280 \times 6.5443}{4} = 562.555\ A.t/pole$$

**<u>Example (25):</u>**

In a 3-phase synchronous machine, the flux per pole is 0.01 Wb, the rotor speed is 1800 rpm, the stator has 36 slots and rotor has 4 poles.

A) Calculate the frequency and rms value of the induced emf in the coil, if coil sides lie in slots 1 and 9, and the coil has 100 turns.

B) An exactly similar coil is now placed with its coil sides in slots adjacent to the first coil. Find the rms value of the induced emf for series connection.

ϕ = 0.01 Wb,
N = 1800 rpm
S = 36
2P = 4
The frequency ($f$) = N×P/60 = 1800×2/60 = 60 Hz

$\tau_c = 9 - 1 = 8$

$\tau = S/2P = 36/4 = 9$

There is chording by ONE slot

Slot angle $\alpha = 180/9 = 20°$

Chording angle $\gamma = \alpha = 20°$

Chording factor $K_c = \cos(\gamma/2) = \cos(10) = 0.9848$

Since there is only one coil, then Q = 1 and the distribution factor $K_d = 1$

The winding factor $K_w = K_c \times 1 = 0.9848$

$EMF = 4.44 \times f \times \phi \times N_{ph} \times K_w$

$EMF = 4.44 \times 60 \times 0.01 \times 100 \times 0.9848 = 262.351$ V

B) Now when another coil is placed adjacent to first coil;

The chording factor is unchanged

Now Q = 2 and $N_{ph} = 200$

$$K_d = \frac{sin\left(\frac{Q\alpha}{2}\right)}{Q\ \sin\left(\frac{\alpha}{2}\right)} = \frac{\sin(20)}{2 \times \sin(10)} = 0.9848$$

The winding factor $K_w = K_c \times K_d = (0.9848 \times 0.9848) = 0.96985$

$EMF = 4.44 \times 60 \times 0.01 \times 200 \times 0.96985 = 516.736$ V

**Example (26)**

Show the shape of the total MMF of a 3-phase, 2-layer, 4-pole machine with 3 slots/pole/phase, and coil pitch 77.78%, when the current in phase A is max. +ve.

2P = 4,

Q = 3, then $S = Q \times 6P = 3 \times 12 = 36$ slots

$\tau = S/2P = 36/4 = 9$

$\tau_c = 0.7778 \times 9 = 7$ (Chorded by 2 slots)

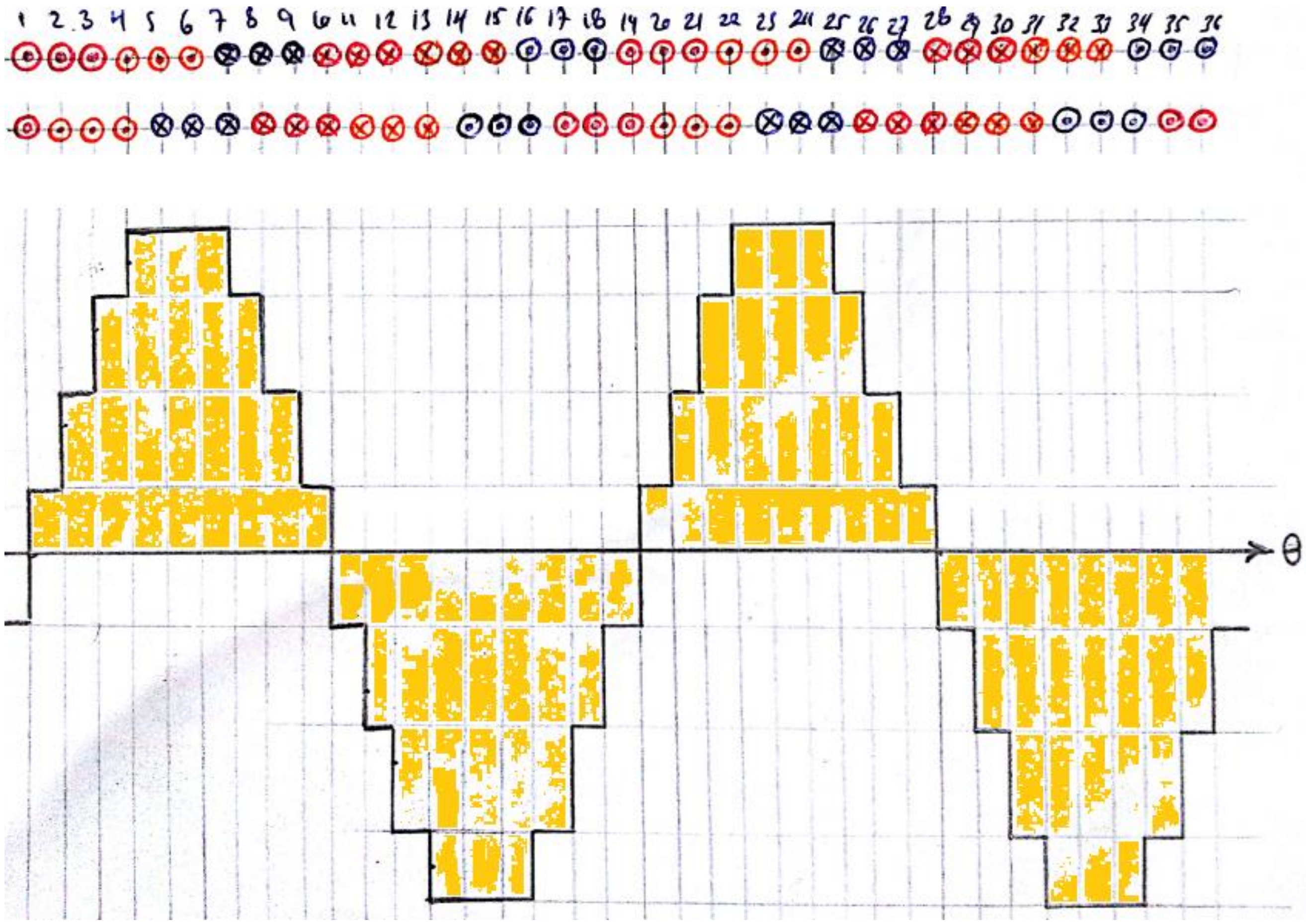
the slot angle $\alpha = 180/9 = 20°$

the phase angle between phases (120°) = 120/20 = 6 slots

Since the current in phase (A) is maximum +ve, then we assume the following symbols

| | |
|---|---|
| Current in phase (A) is max. $\odot$ | MMF produced is $\uparrow$ (1.0 cm) |
| Each Current in phase (B) and (C) is half max $\otimes$ | MMF produced is $\downarrow$ (0.5 cm) |

Based on the above assumption, the total MMF pattern is as given below.



**Example (27)**

Consider a 3-phase, double-layer, synchronous generator, with 18 slots, 6 poles, RYB sequence. Draw the MMF waveform at the instant when the current in phase R is maximum positive in the following two cases and give your comment on MMF wave:

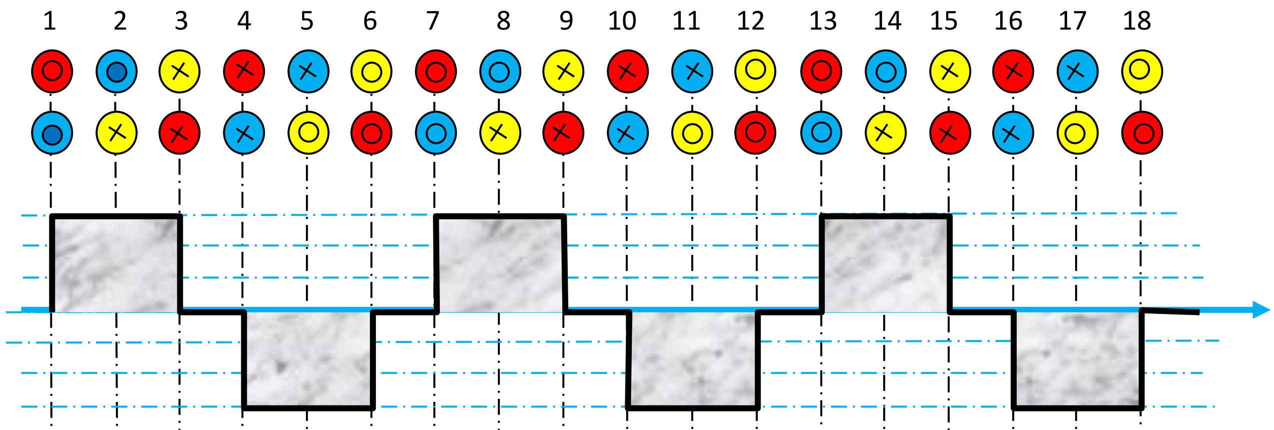
a) If the machine is chorded by one slot b) if the machine is full pitched.

$$\tau = \frac{S}{2P} = \frac{18}{6} = 3, \quad and \quad \tau_c = \tau - 1 = 2$$

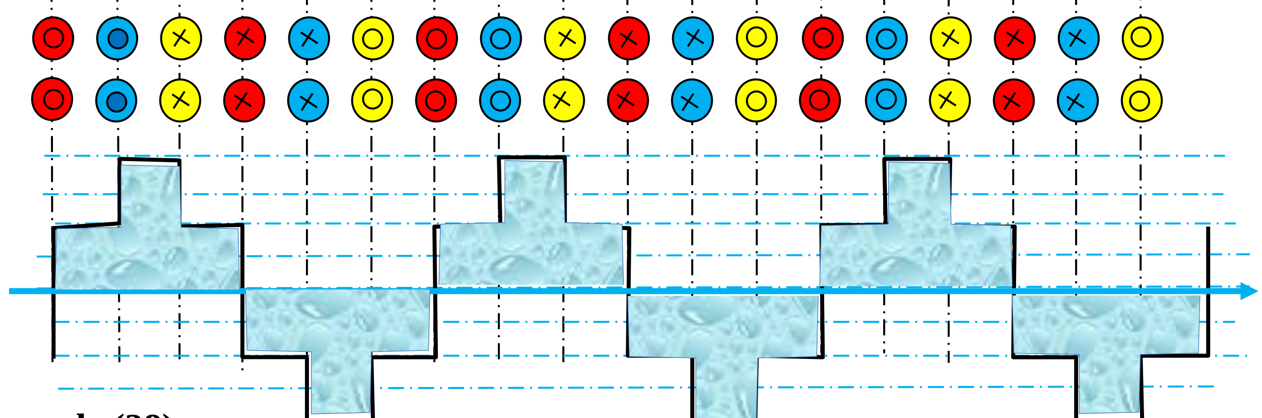
$$Q = \frac{\tau}{3} = 1$$

$$\alpha = \frac{180}{\tau} = \frac{180}{3} = 60° \;\rightarrow\; 120° = 2\,slots$$

a) In case of chording by one slot




b) In case of chording by one slot: **(3.5)MARKS**



**Example (28):**

Consider a 3-phase, double-layer alternator with 30 slots and 4 poles. If the coil span is 7 slots and the phase sequence is RYB, draw the MMF waveform at the instant when the current in phase R is maximum negative.

$$Q = \frac{S}{6\,P} = \frac{30}{6 \times 2} = \frac{5}{2}$$

Since Q is fraction, therefore we construct the following table

| | R | | | | | B | | | | | Y | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| N | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | 8 |
| S | | 9 | | 10 | | 11 | | 12 | | 13 | | 14 | | 15 | |
| N | 16 | | 17 | | 18 | | 19 | | 20 | | 21 | | 22 | | 23 |
| S | | 24 | | 25 | | 26 | | 27 | | 28 | | 29 | | 30 | |

Since the current in phase (A) is maximum -ve, then we assume the following symbols

Current in phase (A) is max. ⊙ MMF produced is ↑ (1.0 cm)

Each Current in phase (B) and (C) is half max ⊗ MMF produced is ↓ (0.5 cm)

The MMF waveform is shown in Fig. 57.

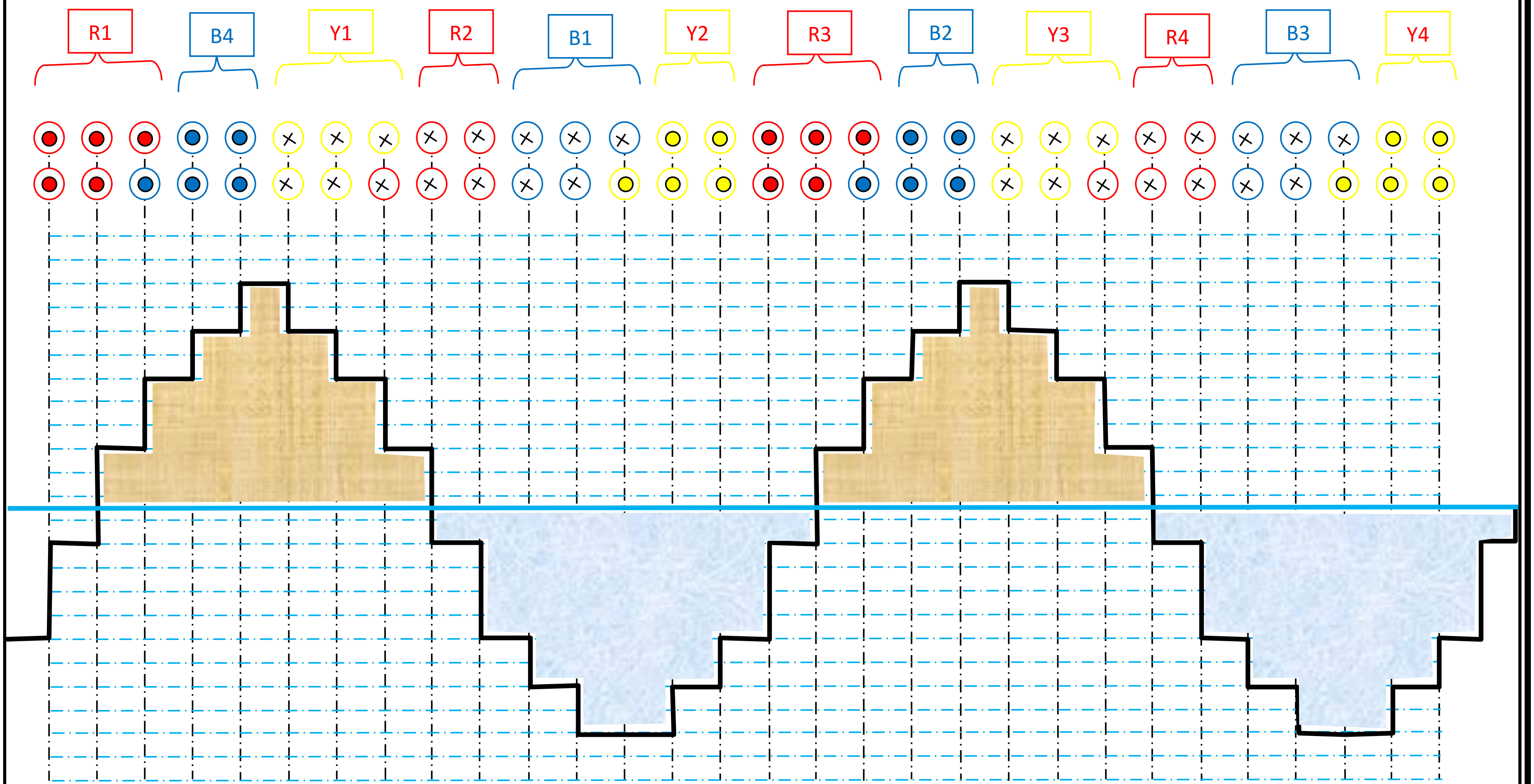
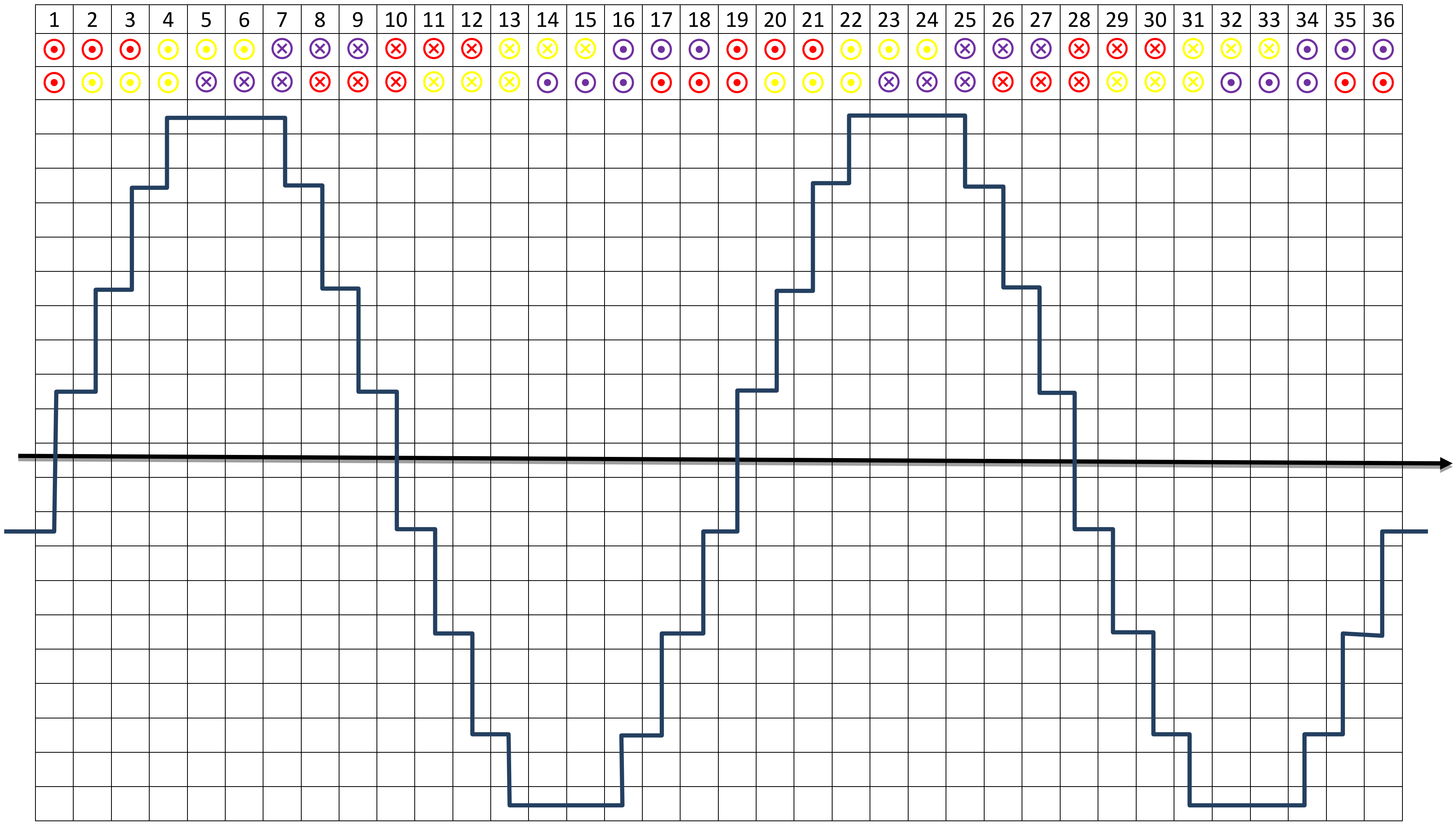


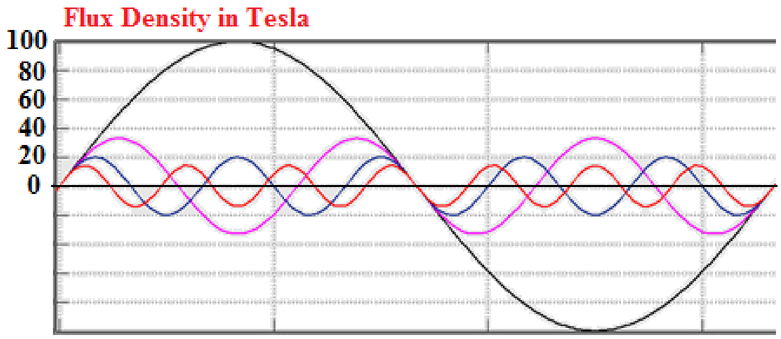

Fig. 57 MMF waveform of 3-phase double layer winding

**Example (29):** Draw the MMF waveform of a 3-phase, 2-layer, 4-pole induction machine with three slots/pole/phase, and coil pitch 77.78%, when the current in phase A is max. +ve.



**Example (30):**

The flux density distribution of a 600 rpm, 3-phase, 10-pole alternator, is represented as shown in figure below.




The pole pitch is 40 cm and core length is 30 cm. The stator coil pitch is chorded by 2 slots such that the coil span is 5/6 of the pole pitch. Determine phase and line values of induced emf if there are 10 conductors per slot in two layers.

From flux density distribution curve:

$B_{1m}$ = 100 Tesla, $B_{3m}$ = 38 Tesla, $B_{5m}$ = 20 Tesla, $B_{7m}$ = 18 Tesla,

$A_1 = 40\times30\times10^{-4} = 0.12\ m^2$

$A_3 = 0.04\ m^2$ and $A_5 = 0.024\ m^2$ and $A_7 = 0.017143\ m^2$

$\phi_1 = B_{1av} \times A_1 = 2/\pi \times 100 \times 0.12 = 7.63944$ Wb

$\phi_3 = B_{3av} \times A_3 = 2/\pi \times 38 \times 0.04 = 0.96766$ Wb

$\phi_5 = B_{5av} \times A_5 = 2/\pi \times 20 \times 0.024 = 0.30558$ Wb

$\phi_7 = B_{7av} \times A_7 = 2/\pi \times 18 \times 0.0.017143 = 0.196444$ Wb

$$f_1 = \frac{600 \times 5}{60} = 50\ Hz$$

$f_3 = 3 \times 50 = 150\ Hz$, $f_5 = 5 \times 50 = 250\ Hz$ and $f_7 = 7 \times 50 = 350\ Hz$

Since $\tau = 180°$ $and\ \tau_c = \frac{5}{6}\tau = \frac{5}{6} \times 180° = 150°$

The chording angle ($\gamma$) = $180° - 150° = 30°$ This is represented by 2 slots

The slot angle ($\alpha$) = 30/2 = 15

Since $\alpha = \frac{180}{\tau} \rightarrow \tau = \frac{180}{15} = 12\ slots \ \rightarrow Q = \frac{\tau}{3} = 4$

$$S = \ \tau \times 2P = 12 \times 10 = 120\ slots$$

$$K_{c1} = cos\left(\frac{\gamma}{2}\right) = \cos\left(\frac{30}{2}\right) = \ 0.96593$$
$$K_{c3} = \left(\frac{3 \times \gamma}{2}\right) \cos\left(\frac{3 \times 30}{2}\right) = \ 0.70711$$
$$K_{c5} = \left(\frac{5 \times \gamma}{2}\right) \cos\left(\frac{5 \times 30}{2}\right) = \ 0.25882$$

$$K_{c7} = \left(\frac{7 \times \gamma}{2}\right) \cos\left(\frac{7 \times 30}{2}\right) = \ -0.25882$$

$$K_{d1} = \frac{sin\left(\frac{Q\alpha}{2}\right)}{Q\ \sin\left(\frac{\alpha}{2}\right)} = \frac{sin\left(\frac{4 \times 15}{2}\right)}{4\ sin\left(\frac{15}{2}\right)} = 0.9577$$

$$K_{d3} = \frac{sin\left(\frac{3Q\alpha}{2}\right)}{Q\ \sin\left(\frac{3\alpha}{2}\right)} = \frac{sin\left(\frac{180}{2}\right)}{4\ sin\left(\frac{45}{2}\right)} = 0.6533$$

$$K_{d5} = \frac{sin\left(\frac{5Q\alpha}{2}\right)}{Q\ \sin\left(\frac{5\alpha}{2}\right)} = \frac{sin\left(\frac{300}{2}\right)}{4\ sin\left(\frac{75}{2}\right)} = 0.20533$$

$$K_{d7} = \frac{sin\left(\frac{7Q\alpha}{2}\right)}{Q\ \sin\left(\frac{7\alpha}{2}\right)} = \frac{sin\left(\frac{420}{2}\right)}{4\ sin\left(\frac{105}{2}\right)} = -0.15756$$

$K_{w1} = K_{c1} \times K_{d1} = 0.92514$

$K_{w3} = K_{c3} \times K_{d3} = 0.461955$

$K_{w5} = K_{c5} \times K_{d5} = 0.05314$

$K_{w7} = K_{c7} \times K_{d7} = 0.04078$

$$N_{ph} = \frac{10 * 120}{2 \times 3} = 200\ turns$$

***Phase voltage***

$E_1 = 4.44\, f_1\, \Phi_1\, N_{ph}\, K_{w1}$ = 4.44 * 50 * 7.63944 * 200 * 0.92514 = 313799.2876 V

$E_3 = 4.44\, f_3\, \Phi_3\, N_{ph}\, K_{w3}$ = 4.44 * 150 * 0.96766 * 200 * 0.461955= 59542.44799 V

$E_5 = 4.44\, f_5\, \Phi_5\, N_{ph}\, K_{w5}$ = 4.44 * 250 * 0.30558* 200 * 0.05314= 3604.951706 V

$E_7 = 4.44\, f_7\, \Phi_7\, N_{ph}\, K_{w7}$ = 4.44 * 350 * 0.196444* 200 * 0.04078= 2489.81455 V

$E_{ph} = \sqrt{E_1^2 + E_3^2 + E_5^2 + E_7^2 + E_9^2} = 319428.3814$

As the machine is ***star connected***, the third harmonic of the line voltage and its odd multiplications (e.g. 9th, 15th, 21th, … etc) become zero

Line voltage = $\sqrt{3}\sqrt{313799.2876^{\,2} + 3604.951706^2 + 2489.81455^2}$ = 543569.281 volt

**Example (31):**

The flux density distribution curve of a 3-phase, synchronous machine is represented at no load by:

$$B = 100 \sin\theta + 21 \sin 3\theta + 15 \sin 5\theta + 7 \sin 7\theta + \sin 9\theta$$

Assuming the fundamental chording factor is 0.96593. The distribution factor is the same as the chording factor for the fundamental and all harmonics.

Determine the induced EMF's at no load by the harmonics as functions of the fundamental.

it is clear that, the pole pitch (area) of the fundamental component is 3 times larger than the pole pitch (area) of the 3rd harmonic
this mean $A_1 = 3\, A_3$

$$\frac{A_1}{A_3} = 3 \;\; or\ in\ general\ \frac{A_1}{A_n} = n$$

Also, $\phi_1 = B_1 \times A_1$ and $\phi_3 = B_3 \times A_3$

$$\frac{\phi_1}{\phi_3} = \frac{B_1}{B_3}\frac{A_1}{A_3} = \frac{B_1}{B_3} \times 3$$

In general:

$$\frac{\phi_1}{\phi_n} = \frac{B_1}{B_n} \times n$$

$$\frac{\phi_n}{\phi_1} = \frac{B_n}{B_1} \times \frac{1}{n} \qquad (1)$$

Also, the relation between the fundamental frequency and harmonic frequency is given by:

$$f_n = n\, f_1$$

This means

$$\frac{f_n}{f_1} = n \qquad (2)$$

The fundamental EMF is given by:

$E_1 = 4.44\, f_1\, \Phi_1\, N_{ph}\, K_{w1}$

The harmonic EMF is given by:

$E_n = 4.44\, f_n\, \Phi_n\, N_{ph}\, K_{wn}$

By dividing $E_n$ over $E_1$, we obtain that

$$\frac{E_n}{E_1} = \frac{f_n}{f_1} \times \frac{\Phi_n}{\Phi_1} \times \frac{K_{wn}}{K_{w1}} \qquad (3)$$

Substituting from (1) and (2) in (3)

$$\frac{E_n}{E_1} = n \times \frac{B_n}{B_1} \times \frac{1}{n} \times \frac{K_{wn}}{K_{w1}}$$

Then

$$\frac{E_n}{E_1} = \frac{B_n}{B_1} \times \frac{K_{wn}}{K_{w1}} \qquad (4)$$

Now we need to calculate the winding factors for the fundamental as well as harmonics Since the chording factor is same as the distribution factor, then we calculate the chording factor only.

**For Fundamental**

Given that $K_{c1} = 0.96593$

Then

$$K_{c1} = \cos\left(\frac{\gamma}{2}\right) = 0.96593$$

Then

$$\frac{\gamma}{2} = 15 \text{ This means } \gamma = 30$$

Also $K_{d1} = 0.96593$

$K_{w1} = K_{c1} \times K_{d1} = (0.96593)^2 = 0.933$

**For Third Harmonic**

$$K_{c3} = \cos\left(\frac{3\gamma}{2}\right) = \cos\left(\frac{3 \times 30}{2}\right) = 0.7071$$

Also $K_{d3}$ = 0.0.7071

$K_{w3} = K_{c3} \times K_{d3} = (0.7071)^2 = 0.5$

$$\frac{K_{w3}}{K_{w1}} = \frac{0.5}{0.933} = 0.53591$$

Also the value of

$$\frac{B_3}{B_1} = \frac{21}{100} = 0.21$$

Based on equation (4)

$$\frac{E_3}{E_1} = 0.21 \times 0.53591 = 0.11254$$

**For Fifth Harmonic**

$$K_{c5} = \cos\left(\frac{5\gamma}{2}\right) = \cos\left(\frac{5 \times 30}{2}\right) = 0.25882$$

Also $K_{d5}$ = 0.25882

$K_{w5} = K_{c5} \times K_{d5} = (0.25882)^2 = 0.067$

$$\frac{K_{w5}}{K_{w1}} = \frac{0.067}{0.933} = 0.07181$$

Also the value of

$$\frac{B_5}{B_1} = \frac{15}{100} = 0.15$$

Based on equation (4)

$$\frac{E_5}{E_1} = 0.15 \times 0.07181 = 0.0107715$$

**For Seventh Harmonic**

$$K_{c7} = \cos\left(\frac{7\gamma}{2}\right) = \cos\left(\frac{7 \times 30}{2}\right) = -0.25882$$

Also $K_{d7}$ = - 0.25882

$K_{w7} = K_{c7} \times K_{d7} = (-0.25882)^2 = 0.067$

$$\frac{K_{w7}}{K_{w1}} = \frac{0.067}{0.933} = 0.07181$$

Also the value of

$$\frac{B_7}{B_1} = \frac{7}{100} = 0.07$$

Based on equation (4)

$$\frac{E_7}{E_1} = 0.07 \times 0.07181 = 5.0267 \times 10^{-3}$$

**For Ninth Harmonic**

$$K_{c9} = \cos\left(\frac{9\gamma}{2}\right) = \cos\left(\frac{9 \times 30}{2}\right) = -\,0.7071$$

Also $K_{d9}$ = - 0.0.7071
$K_{w9} = K_{c9} \times K_{d9} = (-\,0.7071)^2 = 0.5$

$$\frac{K_{w9}}{K_{w1}} = \frac{0.5}{0.933} = 0.53591$$

Also the value of

$$\frac{B_9}{B_1} = \frac{1}{100} = 0.01$$

Based on equation (4)

$$\frac{E_9}{E_1} = \ 0.01 \times 0.53591 = 5.3591 \times 10^{-3}$$

**Example (32):**

A 3-phase, 50 Hz, Y-connected alternator has the following data:
Number of poles = 10
Slots per pole per phase = 2
Conductor per slot (double layer) = 4
Coil span = 150° electrical
Flux per pole (fundamental component) = 0.12 Wb
The analysis of the air-gap flux density shows 20% third harmonic, 12% fifth harmonic, 8% seventh harmonic, and 1% ninth harmonic.

a) Find the phase and line value of the induced emf.
b) If the machine delivers 100 kW at p.f. 0.8 lag, calculate the mmf of the fundamental, fifth and seventh harmonic components then indicate their direction.

2p = 10, Q = 2, pole pitch ($\tau$) = Q×3 = 6, slot angle ($\alpha$) = 180/6 = 30

coil span = 150° = 180 - 30, i.e chorded by one slot then $\gamma$ = 30°

S = $\tau$ × 2p = 60

$$\frac{B_3}{B_1} = 0.2 \ \ \& \ \frac{B_5}{B_1} = 0.12 \ \ \& \ \frac{B_7}{B_1} = 0.08$$

$$\phi_n = \ \phi_1 \frac{B_n}{B_1} \times \frac{1}{n}$$

Therefore,

$$\phi_3 = \phi_1 \frac{B_3}{B_1} \times \frac{1}{3} = \frac{0.12 \times 0.2}{3} = 8\, mWb$$

$$\phi_5 = \phi_1 \frac{B_5}{B_1} \times \frac{1}{5} = \frac{0.12 \times 0.12}{5} = 2.88\, mWb$$

$$\phi_7 = \phi_1 \frac{B_7}{B_1} \times \frac{1}{7} = \frac{0.12 \times 0.08}{7} = 1.3714\, mWb$$

$$\phi_9 = \phi_1 \frac{B_9}{B_1} \times \frac{1}{9} = \frac{0.12 \times 0.01}{9} = 0.133\, mWb$$

$f_1$ = 50 Hz, $f_3$ = 150 Hz, $f_5$ = 250 Hz

$f_7$ = 350 Hz, $f_9$ = 450 Hz

Total number of conductors = 60*4 = 240

Total number of turns = 240 / 2 = 120

Number of turns per phase $N_{ph}$ = 120/3 = 40

***The Chording factors***

$$K_{c1} = \cos\left(\frac{\gamma}{2}\right) = \cos\left(\frac{30}{2}\right) = 0.966$$

$$K_{c3} = \cos\left(\frac{3\gamma}{2}\right) = \cos\left(\frac{90}{2}\right) = 0.7071$$

$$K_{c5} = \cos\left(\frac{5\gamma}{2}\right) = \cos\left(\frac{150}{2}\right) = 0.2588$$

$$K_{c7} = \cos\left(\frac{7\gamma}{2}\right) = \cos\left(\frac{210}{2}\right) = -0.2588$$

$$K_{c9} = \cos\left(\frac{9\gamma}{2}\right) = \cos\left(\frac{270}{2}\right) = -0.7071$$

***The distribution factors***

$$K_{d1} = \frac{sin\left(\frac{Q\alpha}{2}\right)}{Q\ \sin\left(\frac{\alpha}{2}\right)} = \frac{sin\left(\frac{2\times 30}{2}\right)}{2\ sin\left(\frac{30}{2}\right)} = 0.966$$

$$K_{d3} = \frac{sin\left(\frac{3Q\alpha}{2}\right)}{Q\ \sin\left(\frac{3\alpha}{2}\right)} = \frac{sin\left(\frac{180}{2}\right)}{2\ sin\left(\frac{90}{2}\right)} = 0.7071$$

$$K_{d5} = \frac{sin\left(\frac{5Q\alpha}{2}\right)}{Q\ \sin\left(\frac{5\alpha}{2}\right)} = \frac{sin\left(\frac{300}{2}\right)}{2\ sin\left(\frac{150}{2}\right)} = 0.2588$$

$$K_{d7} = \frac{sin\left(\frac{7Q\alpha}{2}\right)}{Q\ \sin\left(\frac{7\alpha}{2}\right)} = \frac{sin\left(\frac{420}{2}\right)}{2\ sin\left(\frac{210}{2}\right)} = -0.2588$$

$$K_{d9} = \frac{sin\left(\frac{9Q\alpha}{2}\right)}{Q\ \sin\left(\frac{9\alpha}{2}\right)} = \frac{sin\left(\frac{540}{2}\right)}{2\ sin\left(\frac{270}{2}\right)} = -0.7071$$

$K_{w1} = K_{c1} * K_{d1} = 0.966 * 0.966 = 0.933$

$K_{w3} = K_{c3} * K_{d3} = 0.7071 * 0.7071 = 0.5$

$K_{w5} = K_{c5} * K_{d5} = 0.2588 * 0.2588 = 0.067$

$K_{w7} = K_{c7} * K_{d7} = -0.2588 * -0.2588 = 0.067$

$K_{w9} = K_{c9} * K_{d9} = -0.7071 * -0.7071 = 0.5$

$E_1 = 4.44\ f_1\ \Phi_1\ N_{ph}\ K_{w1} = 4.44 * 50 * 0.12 * 40 * 0.933 = 994.2048$ volt

$E_3 = 4.44\ f_3\ \Phi_3\ N_{ph}\ K_{w3} = 4.44 * 150 * 8\times10^{-3} * 40 * 0.5 = 106.56$ volt

$E_5 = 4.44\ f_5\ \Phi_5\ N_{ph}\ K_{w5} = 4.44 * 250 * 2.88\times10^{-3} * 40 * 0.067 = 8.5674$ volt

$E_7 = 4.44\ f_7\ \Phi_7\ N_{ph}\ K_{w7} = 4.44 * 350 * 1.3714\times10^{-3} * 40 * 0.067 = 5.7115$ volt

$E_9 = 4.44\, f_9\, \Phi_9\, N_{ph}\, K_{w9} \quad = 4.44 * 450 * 0.133\times10^{-3} * 40 * 0.5 = 5.315$ volt

Total phase voltage $E = \sqrt{E_1^2 + E_3^2 + E_5^2 + E_7^2 + E_9^2}$

$E_{ph} = \sqrt{994.2048^2 + 106.56^2 + 8.5674^2 + 5.7115^2 + 5.315^2} = 999.966$ volt

As the machine is ***star connected***, the third harmonic of the line voltage and its odd multiplications (e.g. 9th, 15th, 21th, … etc) become zero

Total line voltage $E = \sqrt{3}\sqrt{E_1^2 + E_5^2 + E_7^2}$

$E_L = \sqrt{3}\sqrt{994.2048^2 + 8.5674^2 + 5.7115^2} = 1722.1056$ volt

Since $P = 3\, V_{ph}\, I_{ph} \cos(\phi)$, $\quad 100{,}000 = 3 * 999.966 * I_{ph} * 0.8$, $\quad I_{ph} = 41.6681$ A

Fundamental component *mmf* ($F_{a1}$)

$$F_{a1} = 1.35\,\frac{K_{w1} N_{ph}\, I_a}{P}$$

$$F_{a1} = 1.35\,\frac{0.933 \times 40 \times 41.6681}{5} = 419.8705\ A.t/pole$$

Fifth harmonic component *mmf* ($F_{a5}$)

$$F_{a5} = 1.35\,\frac{K_{w5} N_{ph}\, I_a}{5P}$$

$$F_{a5} = 1.35\,\frac{0.067 \times 40 \times 41.6681}{25} = 6.0303\ A.t/pole$$

Direction is opposite to the fundamental component

Seventh harmonic component *mmf* ($F_{a7}$)

$$F_{a7} = 1.35\,\frac{K_{w7} N_{ph}\, I_a}{7P}$$

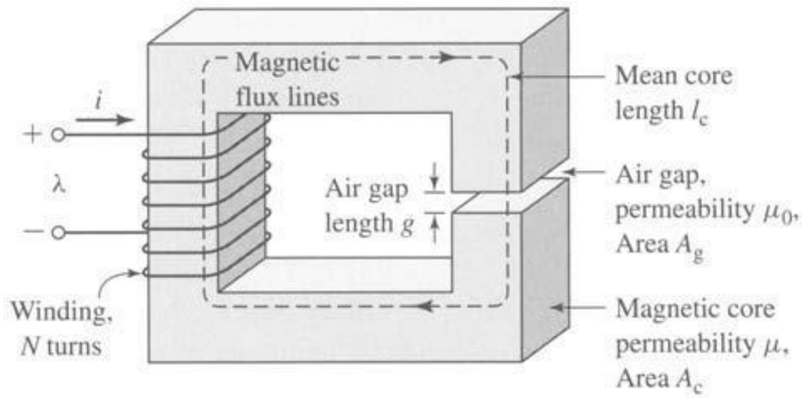
$$F_{a7} = 1.35\,\frac{0.067 \times 40 \times 41.6681}{35} = 4.3074\ A.t/pole$$

Direction is the same as the fundamental component

## Sheet 1 (Magnetic Circuits, 3-ph winding, EMF, and MMF)

1. The magnetic circuit shown in Fig. 1. has dimensions $A_c = A_g = 9\ cm^2$, $g = 0.050$ cm, $l_c = 30$ cm, and N = 500 tums. Assume the value $\mu_r = 70{,}000$ for core material.

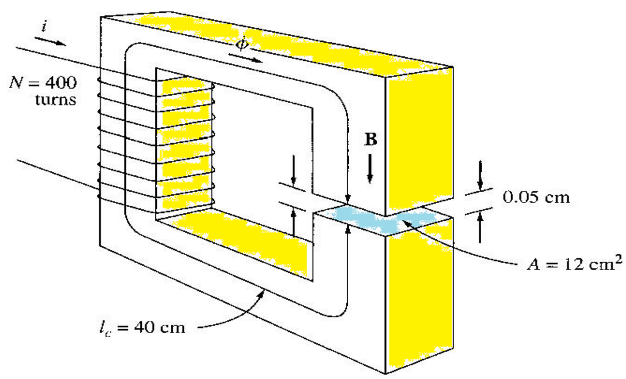
   (a) Find the reluctances $R_c$ and $R_g$. For the condition that the magnetic circuit is operating with $B_c = 1.0$ T, find (b) the flux $\varphi$ and (c) the current i.



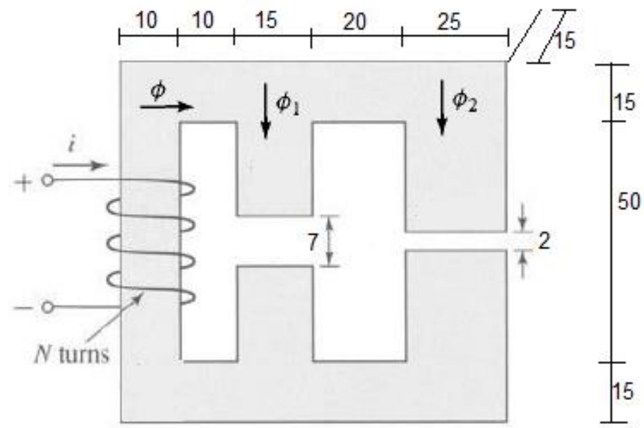

2. A ferromagnetic core with mean path length is 40cm. There is a small gap of 0.05cm in the structure of the otherwise whole core. The CSA of the core is $12cm^2$, the relative permeability of the core is 4000, and the coil of wire on the core has 400 turns. Assume that fringing in the air gap increases the effective CSA of the gap by 5%. Find

   (a) The total reluctance of the flux path (iron plus air gap)

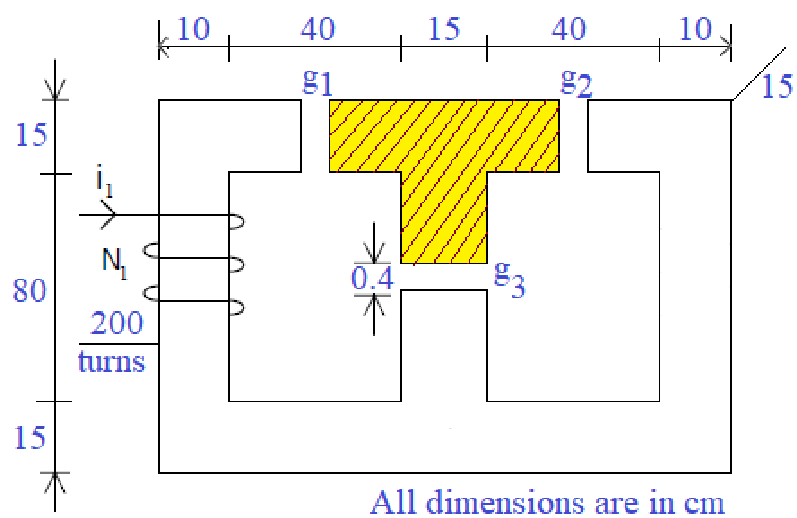
   (b) The current required to produce a flux density of 0.5T in the air gap.



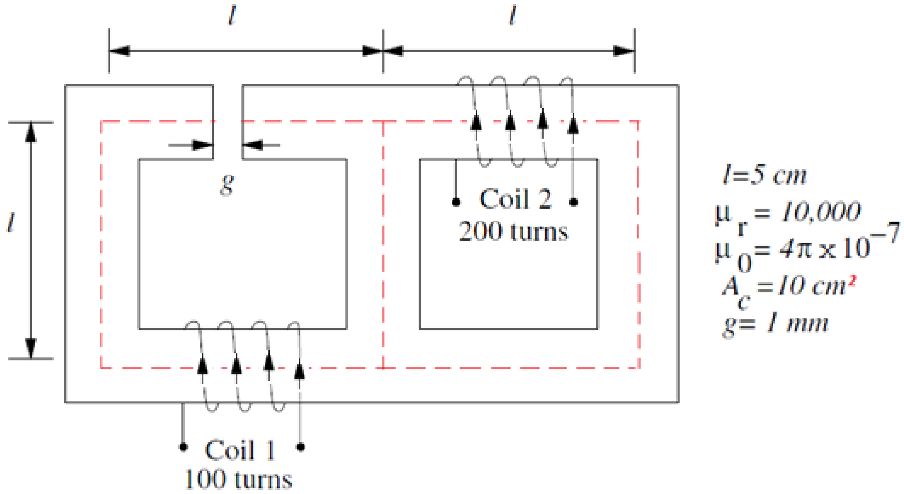

3. A ferromagnetic core with two air gaps with dimensions in centimeter indicated below, the relative permeability of the core is 5000, and the coil of wire on the core has 400 turns and draws a current of 10A. Find the main flux $\phi$ and the flux in each leg $\phi 1$ and $\phi 2$.



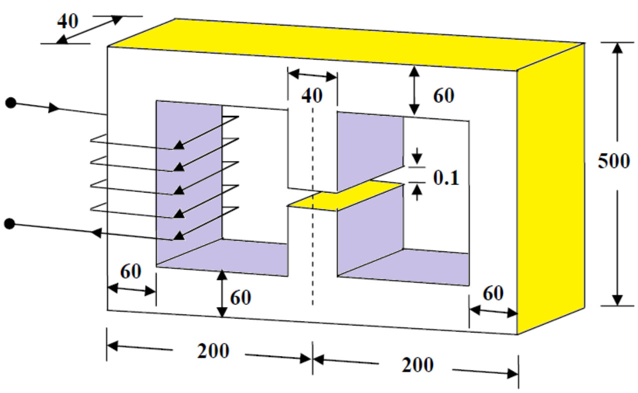

4. The magnetic circuit shown below has three identical air gaps ($g_1$, $g_2$ and $g_3$) and one coil with 200 turns. The circuit is made of two different materials (shaded material has a relative permeability of 1800 but the clear one has a relative permeability of 3000). Calculate the current ($i_1$) to be passed in the coil ($N_1$) to establish a flux density of 0.6 T along the air gap ($g_3$). Consider the fringing effect at the air gap ($g_3$) by 7% and the fringing effect is ignored at the other air gaps.

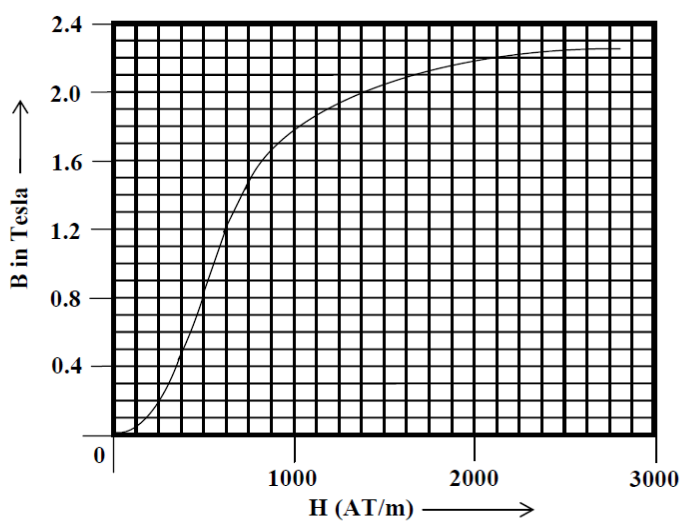



**5.** Two coils are wound on a magnetic core with an air-gap as shown in figure below. Find all of the magnetic fluxes in this magnetic system, assuming that the applied electric currents $i_1 = 4A$ and $i_2 = 2A$.




6. In the magnetic circuit shown in Figure below with all dimensions in mm, calculate the required current to be passed in the coil having 200 turns in order to establish a flux of 1.28 mWb in the air gap. Neglect fringing effect and leakage flux. The B-H curve of the material is given. Permeability of air may be taken as, $\mu_0 = 4\pi \times 10^{-7}$ H/m



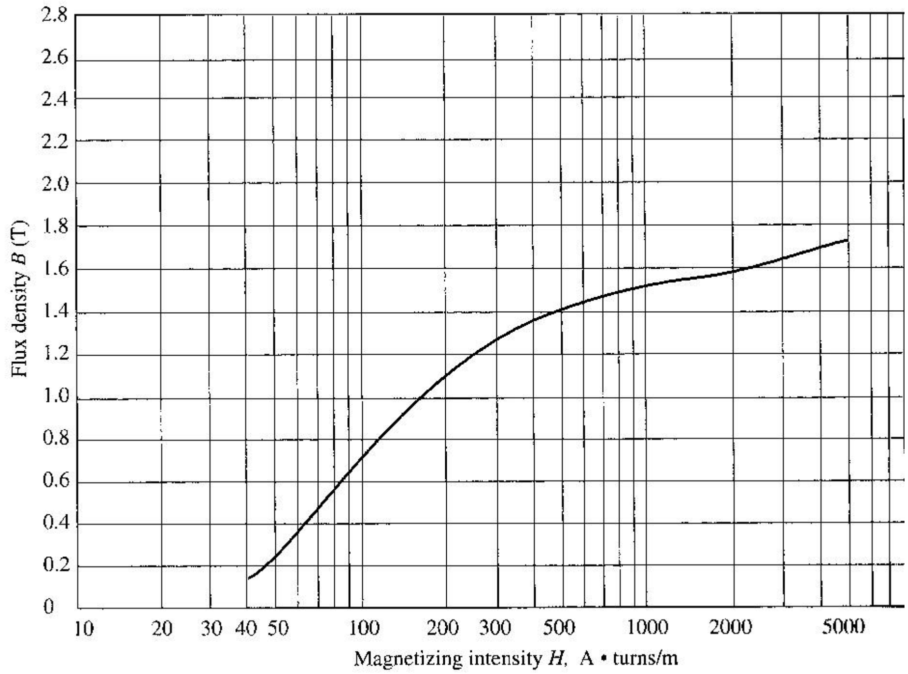




7. A square magnetic core has a mean path length of 55cm and a CSA of 150cm$^2$. A 200 turn coil of wire is wrapped around one leg of the core. The core is made of a material having the magnetization curve shown below. Find:

a) How much current is required to produce 0.012 Wb of flux in the core?

b) What is the core's relative permeability at that current level?

c) What is its reluctance?




8. A 3-phase four-pole generator with 36 slots, Draw the single-layer concentric winding diagram assuming full-pitched coil span. (RYB sequence). Indicate the current direction in each phase based on $T_2$ instant. Then draw the MMF waveform.

9. A 3-phase four-pole generator with 36 slots, Draw the single-layer diamond winding diagram assuming full-pitched coil span. (RYB sequence). Indicate the current direction in each phase based on T3 instant. Then draw the MMF waveform.

10. For a three-phase, double layers 2-pole, 50-Hz alternator with 2 slots per pole per phase. Draw the developed winding diagram assuming full-pitched coil span.(RYB sequence). Indicate the current direction in each phase based on T1 instant. Then draw the MMF waveform.

11. For a three-phase, double layers 2-pole, 50-Hz alternator with 2 slots per pole per phase. Draw the developed winding diagram assuming the coil span is chorded by 1 slot. (RYB sequence). Indicate the current direction in each phase based on T1 instant. Then draw the MMF waveform. Compare between the MMF waveform obtained here with that obtained in the problem (3). What is your comment.?

12. A 3-phasem four-pole generator with 24 slots, Draw the developed winding diagram assuming the coil span is chorded by 2 slots. (RYB sequence)

13. Consider a 3-phase alternator with 27 slots and 6 poles. If the coil span is 4 slots, draw the developed winding of this alternator. (RYB sequence)

14. Consider a 3-phase alternator with 30 slots and 4 poles. If the coil span is 7 slots, draw the developed winding of this alternator. (RBY sequence)

15. A 3-phase, 16-pole alternator has a star-connected winding with 144 slots and 10 conductors per slot. The flux per pole is 0.03 Wb, Sinusoidally distributed and the speed is 375 r.p.m. Find the frequency and the phase and line e.m.f. Assume full-pitched coil.

16. Find the no-load phase and line voltage of a star-connected 3-phase, 6-pole alternator which runs at 1200 rpm, having flux per pole of 0.1 Wb sinusoidally distributed. Its stator has 54 slots having double layer winding. Each coil has 8 turns and the coil is chorded by 1 slot.

17. The stator of a 3-phase, 16-pole alternator has 144 slots and there are 4 conductors per slot connected in two layers and the conductors of each phase are connected in series. If the speed of the alternator is 375 r.p.m., calculate the e.m.f. inducted per phase. Resultant flux in the air-gap is $5 \times 10^{-2}$ webers per pole sinusoidally distributed. Assume the coil span as 150° electrical.

18. A 4-pole, 3-phase, 50-Hz, star-connected alternator has 60 slots, with 4 conductors per slot. Coils are short-pitched by 3 slots. If the phase spread is 60º, find the line voltage induced for a flux per pole of 0.943 Wb distributed sinusoidally in space. All the turns per phase are in series.

19. A 4-pole, 50-Hz, star-connected alternator has 15 slots per pole and each slot has 10 conductors. All the conductors of each phase are connected in series' the winding factor being 0.95. When running on no-load, the terminal e.m.f. was 1825 volt. Calculate the flux per pole.

20. An alternator on open-circuit generates 360 V at 60 Hz when the field current is 3.6 A. Neglecting saturation, determine the open-circuit e.m.f. when the frequency is 40 Hz and the field current is 2.4 A.

21. Calculate the R.M.S. value of the induced e.m.f. per phase of a 10-pole, 3-phase, 50-Hz alternator with 2 slots per pole per phase and 4 conductors per slot in two layers. The coil span is 150°. The flux per pole has a fundamental component of 0.12 Wb and a 20% third component.

22- Show the shape of the total MMF of a 3-phase, 2-layer, lap winding for a 4-pole machine with 3 slots/pole/phase, and having a coil pitch of 77.8% when the current in phase A is (a) Zero (b) maximum positive (c) maximum negative

23- In a 3-phase synchronous machine, the flux per pole is 0.01 Wb, the rotor speed is 1800 rpm. the stator has 36 slots forming 4 poles and having one coil with 100 turns. Calculate the frequency and rms value of the induced emf in the coil if coil sides lie in slots 1 and 9.

An exactly similar coil is placed with its coil sides in slots adjacent to the first coil. Find the emf for the series connection

24- the no-load flux density distribution of a 3-phase, 14-poles, 50 Hz, Y-connected synchronous machine is represented by:

$B = 100 \sin X + 14 \sin 3X + 20 \sin 5X + \sin 7X$

the machine has a stator with 84 slots and double layer windings with 18 conductors per slot and a coil pitch of 5 slots. If the fundamental flux per pole is 0.0212 Wb, determine:

a) $E_3/E_1$ b) $E_5/E_1$

c) $E_7/E_1$ d) the rms phase and line voltages

25- A 3-phase, 50 Hz, Y-connected alternator has the following data:

Number of poles = 10

Slots per pole per phase = 2

Conductor per slot (double layer) = 4

Coil span = 150° electrical

Flux per pole (fundamental component) = 0.12 Wb

The analysis of the air-gap flux density shows 20% third harmonic, 12% fifth harmonic, 8% seventh harmonic, and 1% ninth harmonic.

a) Find the phase and line value of the induced emf.
b) If the machine delivers 100 kW at p.f. 0.8 lag, calculate the mmf of the fundamental, fifth and seventh harmonic components then indicate their direction.

26- The flux density distribution curve of a 3-phase, synchronous machine is represented at no load by:

$$B=100\sin(\theta)+21\sin(3\theta)+15\sin(5\theta)+7\sin(7\theta)+\sin(9\theta)$$

Assuming the fundamental chording factor is 0.96593. The distribution factor is the same as the chording factor for the fundamental and all harmonics.

Determine the induced EMF's at no load by the harmonics as functions of the fundamental.